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Modelling seabed ploughing using the material point method

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Abstract

Ploughing of the seabed is needed for the installation of cables and pipelines and is an area of construction likely to increase given plans in the UK and elsewhere for offshore marine energy (wind and tidal). Seabed ploughing is an expensive and sometimes risky operation for which there are limited guidelines as to what the tow force and speed is for an expected trenching profile within a specified ground condition. Most ploughing schemes are designed using semi-empirical approaches, with very few computational tools able to take into account the geometric and material nonlinearity inherited by the ploughing problem. In this paper we describe how the Material Point Method (MPM) is being used as a numerical tool to model seabed ploughing with the aim of providing an improved predictive tool for the future, via an EPSRC-funded research project at Durham and Dundee Universities. Various issues are discussed in the paper including the means of modelling moving essential boundaries and the choice of basis functions, and this new method in MPM is demonstrated on a simple ploughing problem.

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1. Introduction

Trench-ploughing is defined as the process by which soil is disturbed and displaced during trench formation. This process has been used by mankind for thousands of years for agricultural purposes, but more recently for the creation of trenches to accommodate cables and pipelines on the seabed. Offshore energy generation is expanding worldwide, both in terms of hydrocarbon extraction and renewable energy where trench-ploughing is necessary to accommodate cables and pipelines that transport resources from these offshore sources to the users.

The need for improved methods of designing schemes of seabed ploughing is clear. The UK's Renewable Advisory Board states on behalf of the offshore wind sector, 'To date, installation of subsea cables has been characterised by cost overruns and in a number of cases, cable damage. Subsea cables have been the largest source of insurance claims in offshore wind to date. There is certainly room for savings due to more holistic consideration of cable design and

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installation requirements and methods'. This statement is supported by noting that the installation cost of subsea cables is in the region of 9% of the overall project cost, of which only 0.8% is considered as a material cost [18].

By understanding the mechanical and hydraulic behaviour of soils affected by seabed ploughing, the tow force, velocity and geometry of the plough can be better predicted and designs optimised, and in addition the stability of the plough and trench can be determined. Current understanding is based on semi-empirical approaches that determine the horizontal tow force for pipeline and cable ploughs in both cohesive and cohesionless soils [6]. While 1g-testing has to date largely confirmed these approaches to be acceptable [3,13], more recent studies show a lack of understanding of issues such as temporary suction ahead of the plough, which influences the dynamic response [16,26]. As part of this project, further 1g- and 50g-tests are being undertaken at the University of Dundee to validate the numerical modelling, and some further comments on this aspect of the work are given in §2.

The main aim of this project is to provide a numerical tool able to simulate seabed ploughing, to predict key parameters such as towing force, speed of ploughing and trench stability. First attempts to model tillage cutting in soil using finite element methods (FEM) appeared in the 1990s, but these were often characterised by inadequate soil models (e.g. Duncan & Chang elasticity [7,20]). More promising attempts are reported in [16] using the 2D FEM, and in [19] using the 3D FEM, where a 'Hardening Soil' and complex elastoplastic soil model were used respectively, while updated Lagrangian formulation was used for both 2D and 3D. Other more unusual ideas have surfaced in [11] for instance, where the soil is modelled as a fluid using CFD. In recent years the geotechnical community in particular has shown considerable interest in the Material Point Method (MPM), a mixed Lagrangian-Eulerian approach where very large soil deformation can be modelled. Key examples of the recent use of MPM can be found in [5], for slope stability in [1,2,5,25,27] and foundation modelling [23].

In §3, the approach to model seabed ploughing in this work will be described in detail. One major challenge was the mean of imposing inclined rollers essential boundary conditions to model the soil-plough share interaction. If the problem domain boundaries do not fit the background grid used in the MPM, one cannot make use of the standard FEM approach to impose boundary conditions. By adopting an "implicit boundary" approach, first reported for non-conforming meshes in the FEM [12], this problem has been resolved and arbitrary homogeneous and non-homogeneous essential boundary conditions of any shape and orientation now can be imposed on a regular MPM background grid [8,9].

2. Details of the experimental research

In this section, an overview of the physical experiments being conducted at the Dundee University is discussed. While physical modelling of cable and pipeline ploughing has been conducted previously [3,13,14], further studies are necessary to study scaling effects and to consider uncertainties surrounding the impact of low effective stress in 1g-testing. In this series of tests, cable, pipeline and simplified geometry ploughs (as shown in Figure 1) are being used. The pipeline plough has a fixed share geometry, while the skid angles are adjusted to examine the impact of varying trench depths on the plough performance. Pipeline ploughing is examined at 1g as well as in the Dundee's 3m radius beam centrifuge at 50g, and by comparison of these tests, scaling effects are studied. The cable plough consists of a long narrow geometry where the leading edge is pointed and inclined, while the simplified plough has a vertical rectangular face. To date, the simplified geometry plough has been examined at 1g using three different widths, three different embedded depths and three different soil densities, in order to consider the impact of varying geometries on plough resistance.

In the 1g tests, the simplified geometry ploughs are pre-embedded and are only allowed to move horizontally, while the cable and pipeline ploughs are allowed to move horizontally and vertically in both the 1g and 50g tests. Displacements are measured in either horizontal or both horizontal and vertical directions respectively. Horizontal forces are measured in all tests, while vertical forces are also measured in the simplified geometry plough tests. This allows tow force-displacement relationship to be obtained and the soil surface deformation is scanned efficiently in 3D up to a resolution of $\pm 0.5\text{mm}$ using the methodology proposed in [22]. In the future this data will be used to validate the numerical results obtained by the methodology described in the §3.

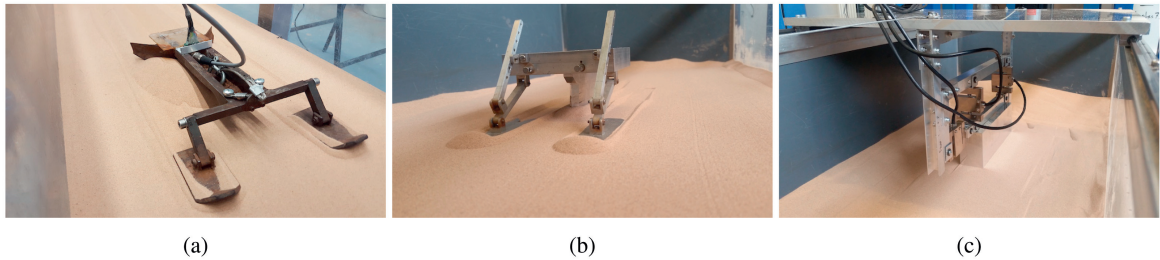


Fig. 1: Types of ploughs. (a) Pipeline plough; (b) Cable plough; (c) Simplified geometry plough.

3. Numerical aspects

3.1. Modelling seabed ploughing using the MPM

The MPM is a semi-meshless method by which the soil is discretised into a set of material points and associated volumes (or masses) while the constitutive equations are solved on a regular background grid of finite elements. The MPM was originally described in explicit form [24] but more recently implicit versions have been gaining popularity [17], because it allows to easily transfer FEA technology into the MPM algorithm. While geotechnical problems have been successfully modelled using the explicit form of the MPM, many studies have avoided issues associated with problem domain boundaries being non-coincident to a background grid boundary. Only recently has the implicit MPM been combined with a method used in the FEM to impose essential boundary conditions onto a non-conforming background grid [12], some details of which will be discussed in the next section, which can provide inclined fixed, roller and prescribed essential boundary conditions in the MPM [8,9]. To explain this issue, if we consider a 2D plan of a pointed plough embedded in soil (as shown in Figure 2), the sides of the share can be initially assumed as frictionless surfaces (under the assumption that tool-soil friction has no significant effect on the soil behaviour) and therefore this can be defined as inclined roller essential boundary conditions. Unless the plough face can be aligned with the background grid, it is not possible to impose this boundary condition as in the standard FEM, hence the need for a special approach where the boundaries do not coincide with the background grid. This approach is simple and avoids the use of contact models to simulate the plough-soil interaction. This approach also allows modelling of ploughing by fixing the plough in space and translating the the soil towards the plough instead, as shown in Figure 2.

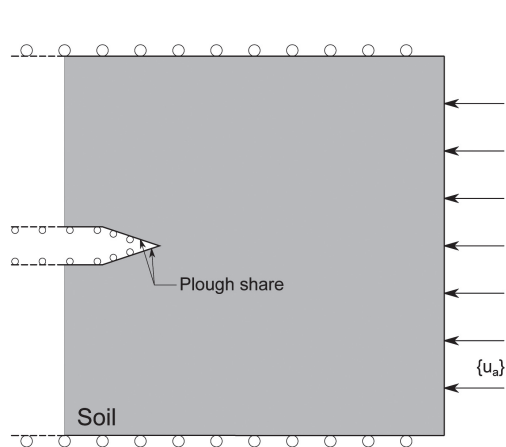


Fig. 2: 2D plough model.

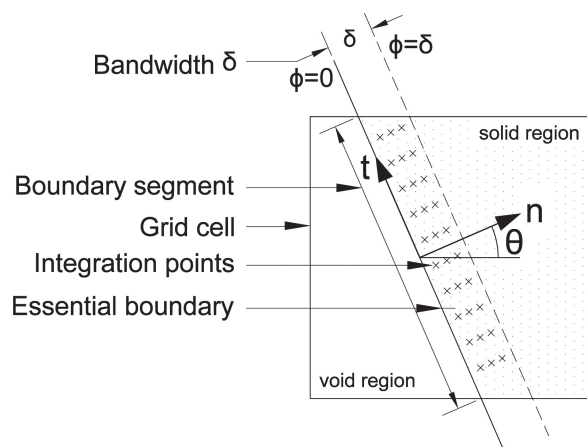


Fig. 3: Essential boundary bandwidth δ through grid element.

3.2. An introduction to the inclined implicit boundary method

In this section, a summary of the principles behind and implementation of non-conforming essential boundary conditions in the MPM is discussed. A regular background grid in the MPM is beneficial as it provides efficient implementation, and maintained the same for each load or time step (unless of course material points move outside the original grid limit, when additional regular elements can be generated). If the background grid has to fit a boundary condition, this imposes an extra burden of mesh generation which can be avoided in the MPM. In similar problems, where a non-conforming mesh in the FEM is used [12], a penalty type approach original developed by Kantorovich and Krylov in the 1950s [10] was used to impose essential boundary conditions on the background grid. This “implicit boundary” method (IBM) introduces penalty stiffness contributions computed for the essential boundary conditions which are added to the element stiffness matrix $[k^E]$. Therefore $[k^E]$ is expanded to include the standard FE stiffness matrix $[K_1]$, and the penalty stiffness matrices $[K_2]$ and $[K_3]$ as follows

$$[k^E] = [K_1] + ([K_2] + [K_2]^T) + [K_3] \quad (1)$$

where $[K_1]$ is computed for every grid element containing material points, and evaluated by summing and weighting the contribution over every material point as follows

$$[K_1] = \int_{V_e} [B_1]^T [D^e] [B_1] dV = \sum_{i=1}^{n_{mp}} [B_1]^T [D^e] [B_1] \det(J_i) v_i \quad (2)$$

where $[D^e]$ is the material constitutive matrix, $[B_1]$ is the standard strain-displacement matrix, J is the Jacobian transformation matrix, v_i is the volume of the material point i and n_{mp} is the total number of material points in the grid element. $[K_2]$ and $[K_3]$ are computed for every grid element which is intersected by an essential boundary condition and evaluated by integrating along the boundary (t -direction) and across the boundary's bandwidth δ (n -direction) as shown in Figure 3, as

$$[K_2] = \int_t [\bar{B}_1]^T \left(\int_0^\delta [\bar{D}_1]^T [D^e] [\bar{D}_2] [T] dn \right) [\bar{B}_2] dt \quad (3)$$

$$[K_3] = \int_t [\bar{B}_2]^T \left(\int_0^\delta [T]^T [\bar{D}_2]^T [D^e] [\bar{D}_2] [T] dn \right) [\bar{B}_2] dt \quad (4)$$

where $[\bar{B}_2]$ and $[\bar{B}_1]$ contain the shape functions and their derivatives respectively, $[\bar{D}_1]$ and $[\bar{D}_2]$ contain the Dirichlet functions and their derivatives respectively and $[T]$ is a rotation matrix for angle θ shown in Figure 3 to achieve the necessary rotation between the n - t axis and local element axis. Prescribed displacements are imposed on grid elements by computing the equivalent element reaction force $\{f_R^E\}$ as

$$\{f_R^E\} = ([K_2] + [K_2]^T + [K_3]) \{u_a\} \quad (5)$$

where $\{u_a\}$ is the prescribed displacement imposed on the grid element nodes. $\{f_R^E\}$ is assembled into the global system of equations $[k]\{u\} = \{f^R\}$ for which displacement $\{u\}$ is solved. Roller boundary conditions are enforced by considering the Dirichlet function in the t direction to be equal to one. A full explanation including derivation of all terms is given in [9].

3.3. Verification of the implicit boundary approach in the MPM

A benchmark test was conducted to verify the implementation of the implicit boundary method in the MPM using inclined rollers and prescribed essential boundary conditions. A 100×100 mm block at two different angles of rotation, 0° and 45° with roller boundary conditions on three sides and a 1mm prescribed displacement on the other side was used, as shown in Figures 4(a-b). A 2D plane strain linear elastic constitutive material with a Young's modulus of 1 GPa and Poisson's ratio of 0.3 was used. A single load step was analysed, and displacement/stress errors (r_u and r_σ respectively) were computed as follows

$$r_u = \sqrt{\frac{\{\{u\} - \{u_{exact}\}\}^T \{\{u\} - \{u_{exact}\}\}}{\{u_{exact}\}^T \{u_{exact}\}}} \quad (6)$$

$$r_\sigma = \sqrt{\frac{\{\{\sigma\} - \{\sigma_{exact}\}\}^T \{\{\sigma\} - \{\sigma_{exact}\}\}}{\{\sigma_{exact}\}^T \{\sigma_{exact}\}}} \quad (7)$$

where $\{u\}$ and $\{u_{exact}\}$ are the numerical and analytical displacement vectors, and similarly $\{\sigma\}$ and $\{\sigma_{exact}\}$ are the numerical and analytical stress vectors in either the xx or yy components denoted as $r_{\sigma_{xx}}$ and $r_{\sigma_{yy}}$ in Table 1 and computed piecewise for every material point as shown in the contour plots (Figures 5(a-c)). As the bandwidth δ is refined, the solution converges to the analytical solution in both 0° and 45° models as shown in Figure 6. Convergence was also observed as the background grid is refined using a fixed $\delta = 1 \times 10^{-5}$. This test demonstrates that inclined roller and prescribed essential boundary conditions can be applied to the MPM irrespective of the grid dimensions and orientation of the boundary conditions. After verifying that the implicit boundary method can lead to accurate imposition of fixed, roller and prescribed essential boundary conditions in the MPM, the second step of the project is to use this method to define the boundary conditions on a plough geometry. If we consider that tool-soil friction has minimal effect on the soil behaviour, the plough share, skids and mouldboards can be defined as roller essential boundary conditions, in the first instance. A 2D simple plough model is presented by defining the plough geometry by a set of roller essential boundary conditions, the soil is defined as linear elastic material (Young's Modulus 1 GPa and Poisson's ratio 0.3) which is pushed by an incremental displacement of $\{u_a\} = 1$ mm for 25 load steps. The problem is solved on a background grid with 10×10 mm grid elements having one material point in each element. While the material parameters have no physical representation and the overall model has no physical meaning, this model is used to illustrate the free movement of the soil along the plough share as shown in Figure 8 from which the soil deformation can be studied. Work is currently ongoing to extend this to three dimensions and to include a realistic material model.

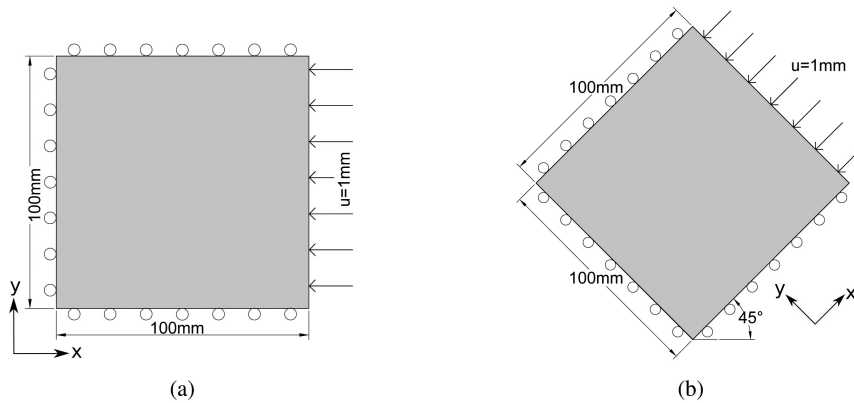


Fig. 4: Contained block under compression. (a) 0° model; (b) 45° model.

Table 1: Displacement and stress errors for 0° , 30° and 45° models with $\delta = 1 \times 10^{-5}$ and $h = 10$ mm.

Sample	Displacement Error r_u	Stress Error $r_{\sigma_{xx}}$	Stress Error $r_{\sigma_{yy}}$
0° Model	4.587×10^{-7}	1.621×10^{-6}	9.697×10^{-7}
45° Model	4.822×10^{-7}	2.636×10^{-5}	2.156×10^{-5}

4. Conclusion

In this paper it is shown how the MPM will be used to model seabed ploughing in 2D and how it will be validated against the experimental data generated at the University of Dundee. Furthermore the deformed models generated from the MPM can be compared with the 3D scans taken for the physical experiments as described in [22]. Roller essential boundary conditions will be used to define the boundaries of the plough geometry assuming that the tool-soil

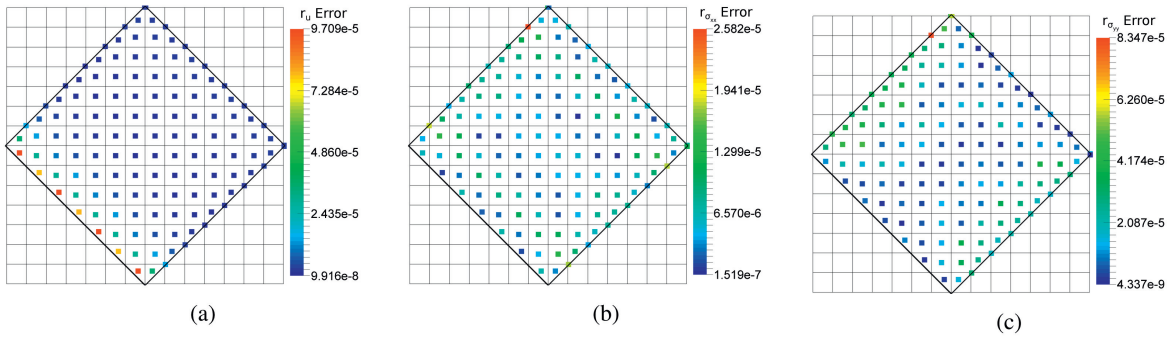


Fig. 5: Error contour plot for the 45° model. (a) displacement r_u error; (b) stress $r_{\sigma_{xx}}$ error; (c) stress $r_{\sigma_{yy}}$ error.

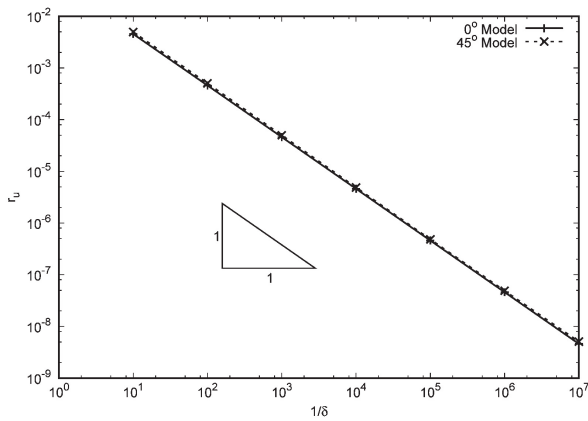


Fig. 6: Displacement convergence with bandwidth δ refinement.

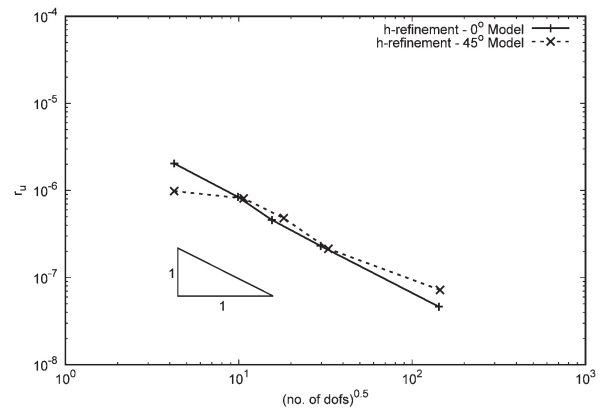


Fig. 7: Displacement convergence with increase of number of dofs.

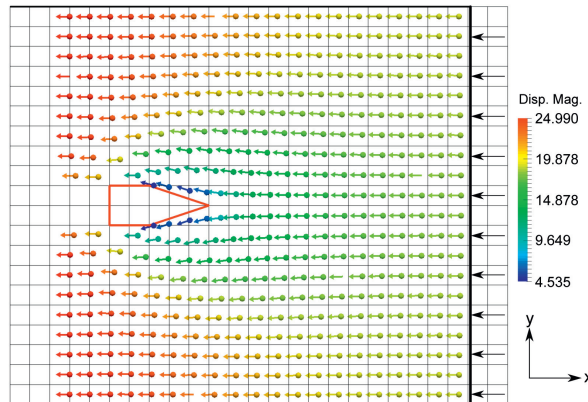


Fig. 8: Displacement vectors at material points.

friction does not contribute significantly to the overall mechanical behaviour of the soil. This is clearly a simplification and ongoing work will take into account any significant additional friction induced along the boundary. While in this paper 2D IBM in the MPM was demonstrated, future work is focusing on delivering 3D IBM for the MPM as 2D simplification of seabed plough modelling is not sufficient [15] and one should consider the 3D effects on the ploughing process. This means that robust and efficient MPM software is necessary to deal with millions of material points

during the discretisation of any complex plough geometry and the imposition of roller essential boundary conditions that defines the external surface of the plough.

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References

- [1] S.M. Andersen and L.V. Andersen, Material-Point-Method analysis of collapsing slopes, *Computational Geomechanics (COMGEO I)*, 817-828, 2009.
- [2] S. Bandara, K. Soga, Coupling of soil deformation and pore fluid flow using material point method, *Computers and Geotechnics*, 63, 199-214, 2015.
- [3] M.F. Bransby, M.J. Brown, A. Hatherley and K. Lauder, Pipeline plough performance in sand waves. Part 1: model testing, *Canadian Geotechnical Journal*, 47, 49-64, 2010.
- [4] M.F. Bransby, M.J. Brown, K. Lauder and A. Hatherley, Pipeline plough performance in sand waves. Part 2: kinematic calculation method, *Canadian Geotechnical Journal*, 47, 65-77, 2010.
- [5] J. Burghardt, R. Brannon and J. Guilkey, A nonlocal plasticity formulation for the material point method, *Computer Methods in Applied Mechanics and Engineering*, 55-64, 2012.
- [6] D.N. Cathie and J.F. Wintgens, Pipeline trenching using plows: Performance and geotechnical hazards, *Offshore Technology Conference*, 715-738, 2001.
- [7] L. Chi and R.L. Kushwaha, A non-linear 3-D finite element analysis of soil failure with tillage tools, *Journal of Terramechanics*, 27, 4, 343-366, 1990.
- [8] M. Cortis, W.M. Coombs, C.E. Augarde, Implicit essential boundaries in the Material Point Method. *Proc. 24th UK Conference of the Association for Computational Mechanics in Engineering*. Cardiff, UK, 116-119, 2016.
- [9] M. Cortis, W.M. Coombs, C.E. Augarde, Imposition of essential boundary conditions in the material point method, *Submitted to International Journal for Numerical Methods in Engineering*, 5 July 2016.
- [10] L. Kantorovich and V. Krylov, *Approximate Methods of Higher Analysis*. Interscience Publishers, 1958.
- [11] S. Karmakar, R.L. Kushwaha, Dynamic modeling of soil-tool interaction: An overview from a fluid flow perspective, *Journal of Terramechanics*, 43, 411-425, 2006.
- [12] A.V. Kumar, S. Padmanabham and R. Burla, Implicit boundary method for finite element analysis using non-conforming mesh or grid, *International Journal for Numerical Methods in Engineering*, 74, 1421-1447, 2008.
- [13] K.D. Lauder, M.J. Brown, M.F. Bransby and F. Simon-Soberon, The influence of soil properties on ploughing speed for offshore pipeline installation, *Proceedings of the 6th International Conference in Physical Modelling in geotechnics (ICPMG 06)*, 709-714, 2006.
- [14] K.D. Lauder, M.J. Brown, M.F. Bransby and S. Boyes, The influence of incorporating a forecutter on the performance of offshore pipeline ploughs, *Applied Ocean Research*, 39, 121-130, 2013.
- [15] A.C. Palmer, Speed effects in cutting and ploughing, *Géotechnique*, 49, 3, 285-294, 1999.
- [16] W. Peng and M.F. Bransby, Numerical modelling of soil around offshore pipeline plough shares, *Frontiers in Offshore Geotechnics II*, 877-882, 2011.
- [17] N. Phuong, A. van Tol, A. Elkadi and A. Rohe, Numerical investigation of pile installation effects in sand using material point method. *Computers and Geotechnics*, 73, 58-71, 2016.
- [18] Renewable Advisory Board, Value breakdown for the offshore wind sector, RAB (2010) 0356, 2010.
- [19] N. Renon and P. Montmitonnet, Numerical simulation of soil plowing by the FEM, *Fifth World Congress on Computational Mechanics*, Vienna, July, 2002.
- [20] U.A. Rosa and D. Wulfsch, Constitutive model for high speed tillage using narrow tools, *Journal of Terramechanics*, 39, 221-234, 1999.
- [21] A. Sadeghirad, R.M. Brannon and J. Burghardt, A convected particle domain interpolation technique to extend applicability of the material point method for problems involving massive deformations, *International Journal for Numerical Methods in Engineering*, 86, 1435-1456, 2011.
- [22] S. Robinson, M.J. Brown, A.J. Brennan, M. Cortis, C.E. Augarde, and W.M. Coombs, Development of low cost 3D soil surface scanning for physical modelling, *3rd European Conference on Physical Modelling in Geotechnics (Eurofuge 2016)*, Nantes, France, 2016.
- [23] W.T. Solowski and S.W. Sloan, Evaluation of material point method for use in geotechnics, *International Journal for Numerical and Analytical Methods in Geomechanics*, 39, 685-701, 2015.
- [24] D. Sulsky, Z. Chen, H.L. Schreyer, A particle method for history-dependent materials, *Computer Methods in Applied Mechanics and Engineering*, 118, 179-196, 1994.
- [25] B. Wang, P.J. Vardon, M.A. Hicks and Z. Chen, Development of an implicit material point method for geotechnical applications, *Computers and Geotechnics*, 71, 159-167, 2016.
- [26] D.J. White and D.N. Cathie, Geotechnics for subsea pipelines, *Frontiers in Offshore Geotechnics II*, 87-123, 2011.
- [27] A. Yerro and E. Alonso, The Material Point Method: A promising computational tool in geotechnics, *Proceedings of the 18th International Conference on Soil Mechanics and Geotechnical Engineering*, Paris, 2013.