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Implicit MPM with second-order convected particle domain interpolation

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ABSTRACT

An implicit material point method (MPM) with the second-order convected particle interpolation (CPDI2) is presented in this paper. In the MPM a body is described by a number of Lagrangian material points, at which state variables are stored and tracked. Calculations are then carried out on a background Eulerian computational mesh. A mapping and re-mapping algorithm is employed, to allow the state variables and other information to be mapped back and forth between the material points and background mesh nodes during an analysis. To reduce the error during these mappings, there are several extensions. The latest extension is termed CPDI2, which uses a quadrilateral particle domain to replace a material point. The CPDI2 extension has been implemented explicitly with using regular grids in published papers. This work develops an implicit CPDI2 method with an elasto-plastic material model. The motivation is that an implicit scheme can reduce the computational cost by allowing a large time step, while enforcing the yield condition accurately and increase stability. Both quadrilateral and triangular particle domains are used. An example shows that the use of a triangular particle domain is more flexible than the quadrilateral particle domain.

Key Words: *material point method; convected particle domain interpolation; implicit methods*

1. Introduction

The Material Point Method (MPM) is a numerical method used to simulate massive deformation of solids requiring less effort than the FEM and meshless methods. In the MPM a body is described by a number of Lagrangian material points, at which state variables are stored and tracked. Calculations are then carried out on a background Eulerian computational mesh. A mapping and re-mapping algorithm is employed, to allow the state variables and other information to be mapped back and forth between the material points and background mesh nodes during an analysis. However, errors commonly are caused by the numerical noise inherent in the original MPM, that arises when a material point crosses the boundary between elements of the computational mesh [1].

To reduce this problem, some extensions to the MPM have been proposed by replacing a zero-volume material point with a finite-volume particle domain, including the Generalized Interpolation Material Point (GIMP) method [2], Convected Particle Domain Interpolation (CPDI) [3], and Second-order CPDI (CPDI2) [4]. For a 2D problem, the particle domains are tracked as rectangles in the GIMP method, as parallelograms in the CPDI method, and as quadrilaterals in the CPDI2 method. The CPDI2 method can more accurately track particle domains and their deformation than others. The CPDI2 has been implemented explicitly with a regular grid in published papers [4].

This work develops an implicit CPDI2 method with an elasto-plastic material model. An implicit scheme can reduce the computational cost by allowing a large time step, whilst enforcing the yield condition accurately and leading to a general increase in stability [5]. The implicit nature occurs at two stages in the calculations: in the solver of the nonlinear boundary value problem and in the stress integration algorithm. Both quadrilateral and triangular particle domains are used in this study.

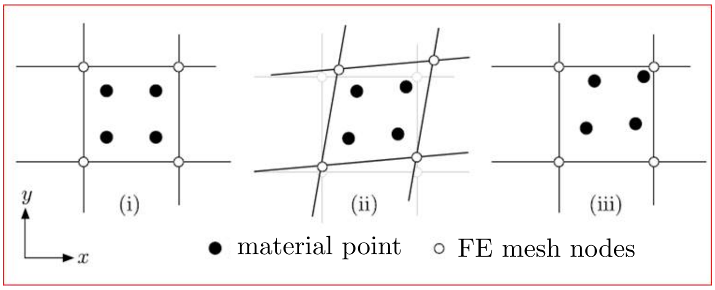

Figure 1: The three phases in one computational step of the material point method: (i) the information held on material points is mapped to the background mesh nodes, (ii) the equilibrium is solved on the mesh to obtain the displacement of the mesh nodes, and (iii) the mesh is reset.

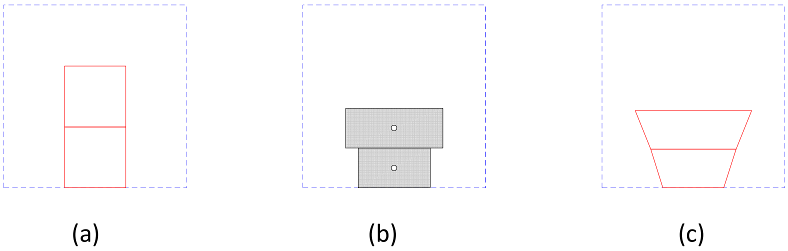

Figure 2: (a) the initial configuration includes two particle domains (solid red) and one element (dashed blue), (b) the deformed configuration computed with the GIMP, (c) the deformed configuration with the CPDI2.

2. Method

2.1. Particle domains

In the GIMP method, a particle domain is a fixed axis-aligned rectangle that translates with the particle, so it may result in a gap or overlap between particle domains, e.g. Figure 2(b). In the CPDI method, a particle domain is a parallelogram, without the edge perpendicularity requirement in the GIMP method. In the CPDI2 method, a particle domain is a quadrilateral, with the coordinates of its four corners stored. These particle domains are similar to a finite element mesh constructed using four-node elements. Therefore, they can exactly track the deformation, e.g. Figure 2(c).

2.2. Material models and implicit implementation

The von-Mises elasto-plastic constitutive law is used here. This law consists of a yield function

$$f(\sigma) = \frac{1}{\rho}\sqrt{2J_2} - 1, \tag{1}$$

where ρ is the yield strength and J_2 the second deviatoric stress invariant. The plastic potential $g = f$.

The implicit backward Euler (bE) integration is used for the stress return when the trial stress enters the plastic regime. Given a trial strain $\epsilon_{\mathbf{t}}$, we need to find the elastic strain $\epsilon^{\mathbf{e}}$ and the plastic multiplier $\Delta\gamma$. The returning stress is directly determined by

$$\sigma^{\mathbf{r}} = [D^e]\epsilon^{\mathbf{e}}, \tag{2}$$

and plastic strain is found from

$$\epsilon^{\mathbf{p}} = \Delta\gamma \left\{\frac{\partial g}{\partial \sigma}\right\}. \tag{3}$$

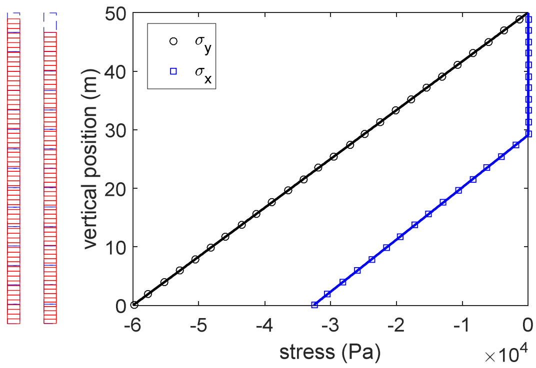

Figure 3: Cauchy stress against the vertical position from the CPDI2 code. The solid lines show the analytical solutions.

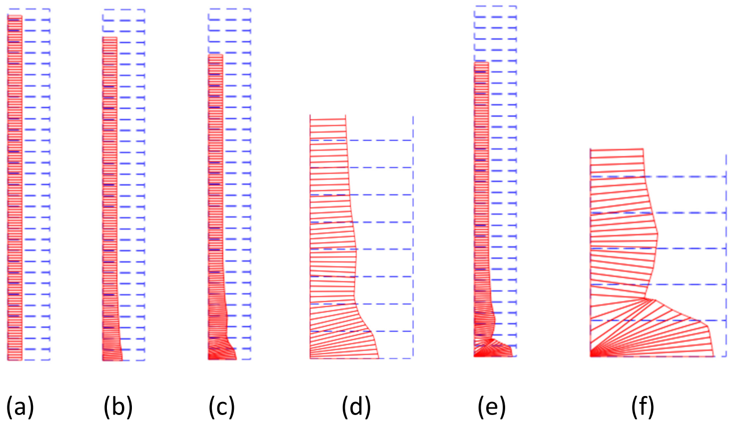

Figure 4: (a) the initial configuration; (b) the deformed configuration subject to a small body force; (c) the deformed configuration when increasing the body force; (d) enlargement of the region at bottom of (c); (e) the deformed configuration with a large body force; (f) enlargement of the region at bottom of (e).

Correspondingly, we have two residuals

$$\text{the yield function} \qquad f = 0, \tag{4}$$

$$\text{balance of strains} \qquad \boldsymbol{\epsilon}^{\mathbf{e}} - \boldsymbol{\epsilon}^{\mathbf{e}}_{\mathbf{t}} + \Delta\gamma \left\{ \frac{\partial g}{\partial \sigma} \right\} = 0. \tag{5}$$

3. Results

An elasto-plastic column subject to self-weight is modelled. In the first example, shown in Figure 3, a roller boundary condition is applied on the both sides of a column of material, hence this problem is equivalent to the 1D problem for which an analytical solution is available. Good agreement is obtained between the numerical and the analytical results validating the approach for this problem.

In the second example, we use this code to model the deformation of column subject to self-weight without side restraint. The roller boundary condition is applied at the left side and the bottom, while the right side is traction-free. As the loading increases, the distortion of particle domains occurs, see Figure 4(f). Because of a very large deformation, a quadrilateral particle domain is degraded to a triangle. This results in the code failing.

In a third example, we use the triangular particle domain CPDI2. With a low body force, we have the deformed configuration in Figure 5(b). With a large body force, we have the deformed

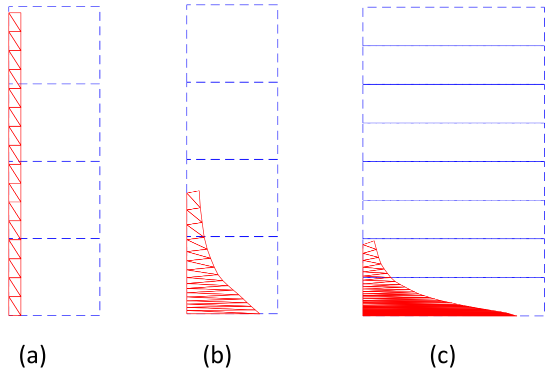

Figure 5: The initial (a) and deformed configurations of a column subject to a low (b) and high (c) self-weight.

configuration in Figure 5(c). The performance of the triangular domain CPDI2 is clearly superior to the quadrialteral case, being more robust when particle domains are severely distorted.

4. Conclusions

We have extended the CPDI2 with quadrilaterial particle domains to use instead triangular particle domains for modelling very large deformation problems. The elasto-plastic material model has been implicitly implemented in the implicit MPM code. In an example, we have shown that the use of triangular particle domains lead to improved stability.

Acknowledgements

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