# ADMM-based Coordinated Decentralized Voltage Control Meets Practical Communication Systems

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***Abstract*—More and more distributed generators (DGs) will penetrate into the distribution power system in the future. Therefore, voltage fluctuation becomes a challenging essential issue for future smart grids. In this paper a coordinated decentralized voltage control method based on a distributed optimal power flow (OPF) algorithm, alternating direction multiplier method (ADMM), is proposed which does not require any control center. Meanwhile, the communication queueing theory for this ADMM is also presented. Simulation results show that the coordinated control approach is sensitive to time-delay in a medium-voltage distribution network (MVDN).**

***Index Terms*—voltage control, coordinated decentralized control, ADMM method, queueing theory, time-delay analysis.**

## I. Introduction

THE reactive power generation in Distributed Generators (DGs) can be adjusted by the electronic inverters to regulate the fluctuation voltage level in distribution network. The voltage control methods can be divided into two categories: centralized optimization control and decentralized optimization control. N. Takahashi *et al.* [1] proposed to use a centralized control center to optimize the reactive power distribution to maintain the voltage level. Such a control system requires communication equipments to send and receive the the control signals between the control center and DG nodes. The time-delay may lead to the voltage out of control during the algorithm operating.

In this context, decentralized optimization control algorithm was proposed to mitigate the effect of time-delay. Distributed OPF problem approaches were first studied in [2], [3], which proposed to divide the transmission network into several areas. Different approaches, such as auxiliary problem principle and alternating direction method, were investigated to work out the distributed OPF problem. In [4], a completely decentralized voltage control approach was proposed without any local information exchange and Genetic Algorithm (GA) was applied to optimize the reactive power distribution. Although the control approach is totally independent, the GA optimization algorithm process is relatively time-consuming compared to real-time control method. More recent distributed algorithms can be found in [5]–[8]. A semidefinite programming (SDP) relaxation technique was applied in the decentralized approaches in [5], [6]. However, this SDP relaxation can only be used into several special networks cases. Other research works in [7], [8] considered ADMM, which is well suited to distributed optimization and in particular to large-scale problems, to optimize the reactive power flow in distribution networks. Both papers can solve the optimization problem without control center. However, the major drawback is that a limited amount of local information communication is still required and the effect of time-delay to the ADMM-based optimization control approach has not been analysed.

The research leading to these results has received funding from the European Commission's Horizon 2020 Framework Programme (H2020) under grant agreement No. 646470, SmarterEMC2 Project. We also acknowledge the support from UK EPSRC grant EP/P005950/1, TOPMOST Project.

In this paper we study the use of coordinated decentralized control approaches building ADMM to solve the OPF problem. When one DG is out of service, the other DGs can still work together and compensate the reactive power to regulate the voltage level. Moreover, the communication time-delay model is presented to analyse the effect of time-delay in practical communication system. Contributions of this paper are summarised as below:

- This paper investigates coordinated decentralized control approaches based on ADMM to solve the OPF problem with limited local information exchange required. This coordinated control method does not require control center and can reduce the optimization time-delay effectively.
- Given the little literature in the research of analysing communication time-delay in smart grid so far, this paper studies a practical communication system with queueing theory and transmission delay to analyse a coordinated optimization voltage control problem.
- The time-delay analysis simulation results show that, with time-delay, existing approaches cannot control the voltage level as expected and thus may affect the system stability.

The rest of this paper is organised as follows. Section II describes the ADMM-Based optimization control approach model, and communication time-delay analysis with queueing delay is introduced in Section III. Simulation results are presented in Section IV. Section V draws conclusions and discusses future work.

## II. ADMM-Based Optimization Control Approach Model

### A. Distribution Network Model

The distributed power flow obeys Ohm's Law and the Kirchoff Laws [9]. Fig. 1 shows the one-line main feeder with a lateral branch distribution network with the notation description. The Distributed power flow equations for a radial distributed network without considering the dashed box are

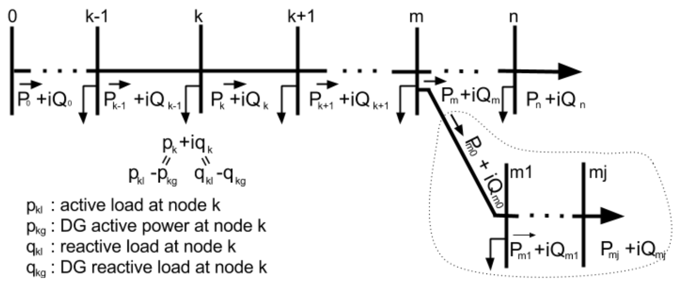


Fig. 1. One-line main feeder with a lateral branching network.

$$P_{k+1} = P_k - r_k \frac{(P_k^2 + Q_k^2)}{V_k^2} - p_{k+1} \tag{1a}$$

$$Q_{k+1} = Q_k - x_k \frac{(P_k^2 + Q_k^2)}{V_k^2} - q_{k+1} \tag{1b}$$

$$V_{k+1}^2 = V_k^2 - 2(r_k P_k + x_k Q_k) + \frac{(r_k^2 + x_k^2)(P_k^2 + Q_k^2)}{V_k^2} \tag{1c}$$

where $P_k$ and $Q_k$ are the active and reactive power flow from node $k$ to node $k+1$, respectively. $V_k$ is the node $k$ voltage magnitude. And $x_k$ and $r_k$ are the resistance and reactance at node $k$, respectively. $p_k + iq_k$ is the total complex power at node $k$. And we have

$$p_k = p_{kl} - p_{kg}, \qquad q_k = q_{kl} - q_{kg} \tag{2}$$

By operating the DG electronic inverter device, only $q_{kg}$ is a controllable parameter. And the inverter can regulate the reactive power to control the voltage level within the operation range. Of course, we also have the following conditions for the substation and terminal node.

$$V_0 = constant, \qquad P_n = Q_n = 0. \tag{3}$$

When a lateral branch is considered, the distributed active and reactive power flow equations for a lateral branch node can be written as

$$P_m + P_{m0} = P_{m-1} - r_m \frac{(P_{m-1}^2 + Q_{m-1}^2)}{V_{m-1}^2} - p_m \tag{4a}$$

$$Q_m + Q_{m0} = Q_{m-1} - x_m \frac{(P_{m-1}^2 + Q_{m-1}^2)}{V_{m-1}^2} - q_m \tag{4b}$$

The new radial feeder formulations can be obtained by the same process with the similar terminal conditions, $V_{m0} = V_m, \quad P_{mj} = Q_{mj} = 0$.

### B. ADMM Optimization Algorithm for Coordinated Control

A coordinated voltage control approach is proposed to regulate the voltage level. The Most Influence Generator (MIG) is proposed based on the sensitivity coefficients [10]. If one DG's reactive power is changed, other neighbouring nodes' voltage level also will change. And the most affected neighbouring DG is called MIG. The steps of the control method are as follows.

1) Check each node's voltage level. If one node's voltage exceeds the operating range, the control method will take action.
2) The number of MIGs will be chosen based on the DG location and network structure.
3) According to the voltage level and numbers of MIGs, this algorithm will calculate the optimization reactive power compensation value of each DG by using the ADMM algorithm and send signal to each MIG to control the voltage.
4) Check the reactive power demand is sufficient to compensate the voltage level. If all DGs' reactive power compensation amount is insufficient, the active power curtailment will be activated to reduce the voltage level.

In this optimization algorithm, the objective function is to minimize the total power losses. The general ADMM method and proof of convergence can be found in [11]. We formulate the objective function to a consensus problem. In a consensus version, each node has its own objective function and constraints which can solve a local optimization problem with local global variables. Because the voltage changes are small compared to the voltage value, the optimization power losses formulation can be approximated to the Linear-DistFlow as

$$min \sum_{k=0}^{n} r_k \frac{Q_k^2}{V_0} \tag{5a}$$

$$\mid Q_{k+1} - Q_k + q_{kl} \mid \leq q_{kmax} \tag{5b}$$

$$V_{k+1} = V_k - 2(r_k P_k + x_k Q_k) \tag{5c}$$

$$V_{min} \leq V_k + \frac{\Delta P_k}{\rho_k} + \frac{\Delta P_{k1}}{\rho_{k1}} + \frac{\Delta P_{k2}}{\rho_{k2}} \leq V_{max} \tag{5d}$$

$$V_1 = 0, \quad Q_n = 0 \tag{5e}$$

where $q_{kmax}$ is the maximum reactive power injection/absorption value for $k$ DG. $\Delta P_k$ and $\rho_k$ are node $k$'s reactive power change and sensitivity coefficient to self node, respectively. $\Delta P_{k1}$ and $\rho_{k1}$ are one MIG's reactive power change and sensitivity coefficient to node $k$, respectively. $\Delta P_{k2}$ and $\rho_{k2}$ are another MIG's reactive power change and sensitivity coefficient to node $k$, respectively. $V_{min}$ and $V_{max}$ are the minimum and maximum operating value for each node.

The distributed ADMM method can be used to solve (5), which the augmented Lagrangian can be given by

$$\begin{aligned}\mathcal{L}_k^{ADMM} =& r_k \frac{Q_k^2}{V_0} + \frac{\rho}{2}(Q_{k+1} - Q_{k+1}^p)^2 + \frac{\rho}{2}(Q_k - Q_k^p)^2 \\ &+ \frac{\rho}{2}(V_{k+1} - V_{k+1}^p)^2 + \frac{\rho}{2}(V_k - V_k^p)^2 \\ &+ y_1(Q_{k+1} - Q_{k+1}^p) + y_2(Q_k - Q_k^p) \\ &+ y_3(V_{k+1} - V_{k+1}^p) + y_4(V_k - V_k^p)\end{aligned} \tag{6}$$

where $Q_k^p$ and $V_k^p$ are the global active and reactive power variables before the new iteration for node $k$, respectively. $y_1, y_2, y_3, y_4$ are the Lagrange multipliers. And $\rho$ is the penalty parameter.

The ADMM consensus method is an iterative algorithm. The $i$th iteration for each node $k$ of the algorithm are updated as follows.

1) Minimise objective function for each node $k$. This step solves equation (6) with constrains (5b), (5c), (5d) and (5e). Each node can optimize its own objective function independently. The minimization results for the $i$th iteration can be used to update the global variables.
2) Update the global variables $Q$ and $V$ for each node. The variables are updated by the following rule:

$$Q_k^p(i+1) = \frac{1}{2}(Q_{k+1}(i+1) + Q_k(i+1)) \tag{7a}$$

$$V_k^p(i+1) = \frac{1}{2}(V_{k+1}(i+1) + V_k(i+1)) \tag{7b}$$

$$Q_n(i+1) = 0, \quad Q_0(i+1) = Q_0(i) \tag{7c}$$

$$V_n(i+1) = V_n(i) \tag{7d}$$

3) Update the Lagrange multipliers for each node. The update rule for each node is given by:

$$y_1(i+1) = y_1(i) + \rho(Q_{k+1}(i+1) - Q_{k+1}^p(i+1)) \tag{8a}$$

$$y_2(i+1) = y_2(i) + \rho(Q_k(i+1) - Q_k^p(i+1)) \tag{8b}$$

$$y_3(i+1) = y_3(i) + \rho(V_{k+1}(i+1) - V_{k+1}^p(i+1)) \tag{8c}$$

$$y_4(i+1) = y_4(i) + \rho(V_k(i+1) - V_k^p(i+1)) \tag{8d}$$

The actual values of reactive power injection/absorption by inverters can be obtained as

$$q_{kg} = Q_{k+1} - Q_k + q_{kl} \tag{9}$$

The performance of the consensus version of the ADMM algorithm will be tested in section IV. Each node will communicate with its neighbouring nodes and MIG nodes to update the local variables in order to find a global optimal solution. To find the effectiveness of the control method, it is necessary to calculate the time-delay in the ADMM algorithm.

## III. Communication Time-Delay Analysis with Queueing Theory

In this section, the M/M/1 queueing system will be adapted to this ADMM communication scenario. This system consists of a single buffer and single server. Meanwhile, packets arrive with arrival rate $\lambda$ following to a Possion process and the service times are exponentially and independently distributed with service rate $\mu$ [12].

According to the probabilistic interpretation, we have

$$N = \sum_{n=0}^{\infty} nP_n = \sum_{n=0}^{\infty} n(1-\rho)\rho^n = \frac{\rho}{1-\rho} \tag{10}$$

where $\rho = \frac{\lambda}{\mu}$, and $N$ is the expected number of packets in the system at steady-state which means the packets both in the waiting buffer and in service. We can also calculate the packet average delay by Little's Theorem

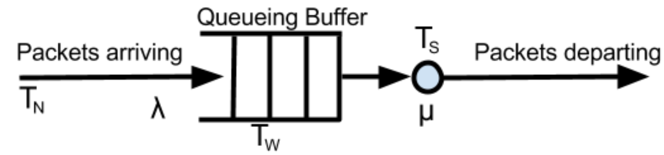


Fig. 2. Communication singal process model.

$$T = \frac{N}{\lambda} = \frac{\rho}{\lambda(1-\rho)} = \frac{1}{\mu-\lambda} \tag{11}$$

$T_N$ is transmission delay between two nodes in Fig.2. According to the 3rd Generation Partnership Project (3GPP) Release 9, the transmission time-delay can be analysed as follows [13].

1) Choose a scenario (Indoor, Micro cellular, Base coverage urban or High speed), and determine the network structure and other parameters, e.g., the number and location of base stations (BSs) and DGs.
2) Assign the propagation condition, e.g., line-of-sight (LOS) or non-LOS (NLOS).
3) Compute the path loss for each BS-DG link in the system.
4) Generate other parameters, e.g., delay spread.
5) Calculate the transmission delay $\tau$. Transmission time-delay is drawn randomly from the delay distribution defined in [13], with an exponential delay distribution in DN scenarios as below:

$$\tau_{i,j}^{'} = -\sigma_{i,j} r_{i,j} ln(X_{i,j}) \tag{12}$$

where $i$ and $j$ are the transmitter index and receiver index, respectively. $\sigma_{i,j}$ is the delay spread, $r_{i,j}$ is the delay distribution proportionality factor, $X_{i,j} \sim$ Uni$(0,1)$ and index $i = 1, ..., N, j = 1, ..., M$. With uniform delay distribution, the time delay values $\tau_{i,j}^{'}$ are drawn from the corresponding range.

The time-delay between two nodes consists of three main delay parts: decision-making delay, transmission delay and queueing delay. For the decision-making delay calculation, ADMM is a suitable choice for a large-scale distributed computing system with less iteration time. To summarize, the communication system delay for each packet is given as follows,

$$T = T_M + T_N + T_W + T_S \tag{13}$$

where $T_M$ is the ADMM algorithm iteration time. $T_N$ is the 3G transmission time-delay. $T_W$ is the packet waiting time in the queue and $T_S$ is the packet service time.

## IV. Simulation Results

In order to verify the coordinated control approach, a 33-bus MVDN is applied. The single line diagram of the distribution network is shown in Fig.3. This system is a 100 KVA, 12.66 KV, radial DN system [14]. It contains 33-bus, 32 branches, four wind DGs(2,12,15,18) and four solar DGs(23,25,27,33).

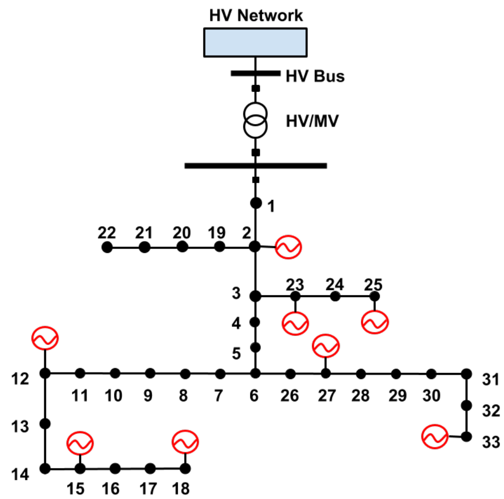


Fig. 3. Generation Power Profile (p.u.).

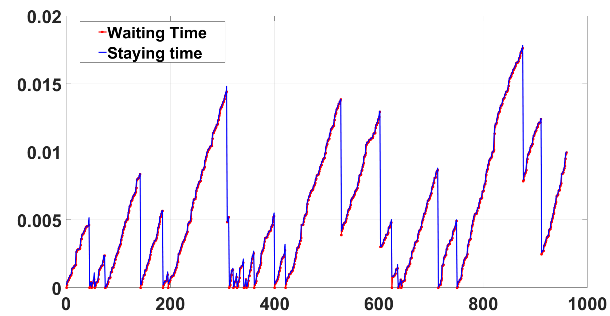


Fig. 5. The waiting time vs staying time in queueing delay.

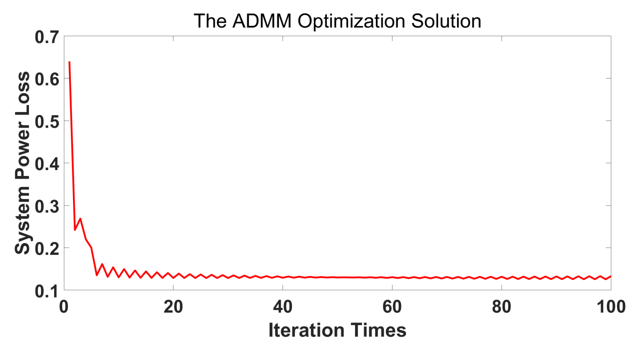


Fig. 6. The ADMM optimal solution.

### A. Queueing Delay Analysis

IEC 61850-9-2 LE [15] specifically designed for MVDN defines the sampling rate for protection applications which is 80 samples per nominal period. And the intelligent electronic device (IED) sampled measured values (SMV) frame is 160B for each packet. In this simulation, we use the Avago technologies' HCTL-2017-A00 as the Decoder IC to calculate the queueing delay.

Fig.4 presents the arriving time of each packet compared to leaving time. Since the service rate is faster than the arriving rate, the packet arriving time is close to the leaving time. The queueing time-delay for each packet can be found in Fig.5. For several points, there are no packets in the queueing buffer, then the waiting time becomes 0 and only service time exists in the system. The results show that the maximum queueing delay can reach 0.018 second which will be larger than the sampling time. Hence, it is necessary to increase the service rate which can reduce the queueing delay within the sampling time. Otherwise, the control algorithm cannot be operated during the sampling time effectively.

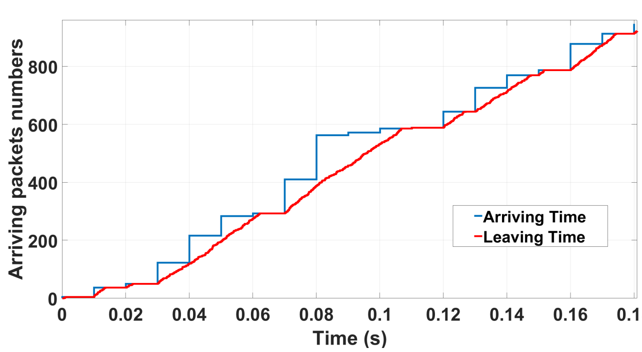


Fig. 4. The arriving time VS leaving time in queueing delay.

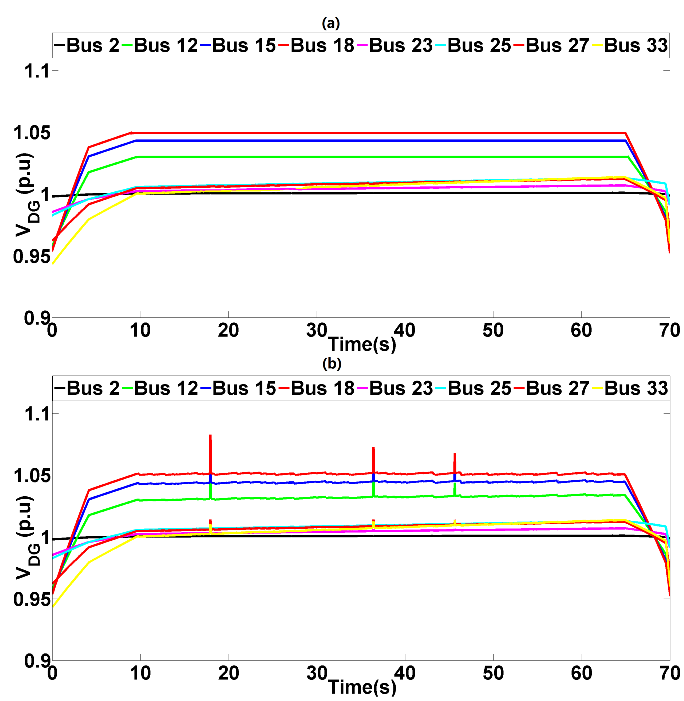


Fig. 7. DGs bus voltage level (a) without time-delay (b) with time-delay.

### B. Voltage control based on ADMM with Time-Delay

Fig. 6 depicts the optimization ADMM solution for this control method. It is obvious that the algorithm can obtain the optimal power loss 0.1288 MW effectively and quickly only after 10 iterations. The ADMM algorithm only needs local node communication which can really reduce the system transmission time-delay. Meanwhile, the convergence of this ADMM algorithm is sensitive to the penalty parameter, choosing too small or large can really influence the rate of convergence.

In order to analyse the reactive power injection/absorption to regulate the voltage level, we will add the time-delay into the control system to analyse the system performance. Fig.7 (a) presents that when the control algorithm does not consider time-delay, the system voltage level can be regulated effectively without going over the maximum operation value. However, if the control algorithm considers the time-delay during the communication of information, the voltage level cannot be regulated and would increase to a high value during the time-delay.

Power system stability could be affected by a voltage collapse lasting from one second to tens of minutes and transient voltage fluctuation is often the main concern [16]. From Fig.7 (b), the voltage level could reach up to 1.08 p.u. during the time-delay. If it happened in a real power system, the power system may be damaged and even blackout without automatically restoring.

## V. Conclusion and Future Work

The high penetration of DGs in MVDN can impact system voltage level. This paper proposed a coordinated decentralized voltage control approach to regulate the DG reactive power injection/absorption. The MIG concept improves the entire independent control approach which may lead to the voltage going out of control if one DG is broken down. Meanwhile, it also could reduce the possibility of active power curtailment and improve the DG power factor. A 33-bus MVDN was used to verify the proposed control method.

The optimization approach based on the ADMM algorithm is applied to the proposed control method which could decrease the existing decision-making time-delay without requiring a control center. ADMM is especially suitable for a large-scale power system when an optimal power flow problem is required.

In this paper, we highlight the communication system in this decentralized control approach. M/M/1 queueing system is applied to analyse the effect of the existing queueing time-delay to the control algorithm. The existing transmission delay calculation is based on a 3G network which has relatively large time-delay values compared to 4G/5G networks. In the future, a more efficient and lower time-delay communication system will be studied to optimize this decentralized voltage control method.

## References

[1] V. Gungor, B. Lu, and G. Hancke, "Opportunities and Challenges of Wireless Sensor Networks in Smart Grid," *IEEE Transactions on Indurtrial Electronics*, vol. 57, no. 10, pp. 3557–3564, Oct 2010.

[2] B. H. Kim and R. Baldick, "Coarse-grained distributed optimal power flow," *IEEE Transactions on Power Systems*, vol. 12, no. 2, pp. 932–939, May 1997.

[3] ——, "A comparison of distributed optimal power flow algorithms," *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 599–604, May 2000.

[4] V. Calderaro, G. Conio, V. Galdi, G. Massa, and A. Piccolo, "Optimal decentralized voltage control for distribution systems with inverter-based distributed generators," *IEEE Transactions on Power Systems*, vol. 29, no. 1, pp. 230–241, Jan 2014.

[5] A. Y. S. Lam, B. Zhang, and D. N. Tse, "Distributed algorithms for optimal power flow problem," in *51st IEEE Conference on Decision and Control (CDC)*, Dec 2012, pp. 430–437.

[6] E. Dall'Anese, H. Zhu, and G. B. Giannakis, "Distributed optimal power flow for smart microgrids," *IEEE Transactions on Smart Grid*, vol. 4, no. 3, pp. 1464–1475, Sept 2013.

[7] T. Erseghe, "Distributed optimal power flow using admm," *IEEE Transactions on Power Systems*, vol. 29, no. 5, pp. 2370–2380, Sept 2014.

[8] P. Sulc, S. Backhaus, and M. Chertkov, "Optimal distributed control of reactive power via the alternating direction method of multipliers," *IEEE Transactions on Energy Conversion*, vol. 29, no. 4, pp. 968–977, Dec 2014.

[9] M. Baran and F. F. Wu, "Optimal sizing of capacitors placed on a radial distribution system," *IEEE Transactions on Power Delivery*, vol. 4, no. 1, pp. 735–743, Jan 1989.

[10] M. Brenna, E. De Berardinis, L. Delli Carpini, F. Foiadelli, P. Paulon, P. Petroni, G. Sapienza, G. Scrosati, and D. Zaninelli, "Automatic Distributed Voltage Control Algorithm in Smart Grids Applications," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 877–885, June 2013.

[11] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations Trends Mach. Learn.*, vol. 3, no. 1, pp. 1–122, 2011.

[12] G. Redinbo, "Queueing systems, volume i: Theory - leonard kleinrock," *IEEE Transactions on Communications*, vol. 25, pp. 178–179, 1977.

[13] P. Kyosti, J. Meinila, and L. Hentila, "WINNER II Channel Models," *European Commission, Deliverable IST-WINNER D1.1.2 ver 1.1*, Sep. 2007.

[14] M. Baran and F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Transactions on Power Delivery*, vol. 4, no. 2, pp. 1401–1407, Apr 1989.

[15] "Communication networks and systems for power utility automation part 9-2: Specific communication service mapping (scsm) sampled values over iso/iec 8802-3. second edition," in *IEC International Standard 61850-9-2,Ed. 2.0*, Nov 2011.

[16] J. Momoh, *Stability Analysis Tools for Smart Grid*. Wiley-IEEE Press, 2012, pp. 51–99.