Power Grid Observability Redundancy Analysis under Communication Constraints

Minglei You, Qitao Liu, Jing Jiang and Hongjian Sun
School of Engineering and Computing Sciences
Durham University, Durham, DH1 3LE, United Kingdom
Email: {minglei.you, qitao.liu, jing.jiang, hongjian.sun}@durham.ac.uk

***Abstract*—The Phasor Measurement Units (PMUs) have largely empowered the current and future smart grid applications, which play important roles in both power systems and Information and Communication Technology (ICT) systems within smart grid. They can provide real-time power grid monitoring and control, which are the fundamentals of various smart grid applications, such as state estimation and demand response. Correspondingly, it also imposes strict time performance requirement on the communication network formed by the PMUs. If the PMU measurements cannot reach the control center within a certain latency bound, it will be invalid for the calculation and may compromise the observability performance of the whole power grid as well as related smart grid applications. In order to provide a statistical guaranteed service from ICT systems to realize critical monitoring and controls, a PMU communication network resource allocation method has been proposed to maximize the overall power grid observability redundancy. Simulation results show that the proposed method can improve the power grid observability redundancy performance with given communication network resources.**

I. Introduction

Smart grid requires the integration of both advanced power grid devices and state-of-art Information and Communication Technology (ICT) systems. To meet such needs, smart devices such as Phasor Measurement Units (PMUs) have been developed and widely applied. With the advanced communication features, PMUs have greatly improved the standards of power system, especially in the aspect of power grid monitoring and control [1]. The PMUs are usually installed at selected buses in the power grid, which can provide measurements of both voltage and current phasor at that bus. The measurements are synchronised via the Global Positioning Satellite (GPS) and transmitted to the control center via the communication network formed by the PMUs [2]. Since one PMU is capable to provide the information of each branch connected to that bus, we can use a relatively smaller number of PMUs to monitor the whole power grid operation status. With these real-time information from PMUs deployed across the power grid, potential applications like real-time stability enhancement and vulnerability assessments have been enabled. To this end, it stimulates various researchers to investigate the optimal PMU locations to observe the entire power system [3], [4].

However, the power grid status monitoring is critical to guarantee the power system operation, hence it is beneficial to maintain certain degrees of observability redundancy in case of PMU failure. There are already methods to keep the whole power system observable for the cases when one or multiple PMUs fail, such as primary and backup method [4] and local redundancy method [3]. On one hand, the real-time power grid monitoring has a stringent time performance requirement. The PMU measurements have to be collected at the control center for the successive data processing within a valid time period. The late arrived measurements will not be valid for usage and may compromise the overall grid monitoring performance. On the other hand, the communication delay cannot be avoided in practical system, especially for the wireless communication systems. Wireless communication systems have many advantages over its wired counterpart, such as low cost, flexibility and scalability. Yet due to the broadcasting nature, wireless communication systems are influenced by communication channels, namely the physical environment. This communication system performance fluctuation will result in the latency experienced by the data packages. However, in the existing literatures, the influence of communication delay on the power system observability performance has not been well addressed, which is the major research problem of this work.

The communication latency is a metric used in the link layer in the Open Systems Interconnection (OSI) model. The composition of the overall communication delay is complex. Some latencies are due to the overheads, which can be treated as fixed latency. Others are varying with time in nature, which is hard to bound or analyse. One major uncertainty contributed to the overall latency comes from the physical layer, where physical communication channel fading is the main reason. An obstacle in modeling the latency with the real-time varying channel parameters is that, latency is not the metric considered in the physical layer. This renders the study of such problems in much complex cross-layer analyses. Another challenge is that, in most fading channel scenarios, it is not feasible to provide a determined bound for the communication delay, which is a consequence of communication performance fluctuation induced by channel fading [5]. To address this research gap, the link layer communication channel model, namely effective capacity theory, has been considered in this paper, which provides a way to estimate the statistical delay bound under fading channel conditions. The effective capacity theory has been widely applied as a powerful quality of service analytical tool in many scenarios [6]. Besides, the effective capacity under various fading scenarios have been well studied and readily available [7], which facilitates the application of this research to various practical situations.

In this work, the aforementioned problems have been studied by simultaneously considering both power grid and communication networks aspects. A communication resource allocation method has been proposed, which aims at maximizing the overall power grid observability redundancy. The proposed method improves the statistical power grid service guarantee via optimizing the resources within ICT systems. Besides, a case study based on the IEEE 14 bus power system has been used to verify the performance of the proposed method, which will be detailed in the later part of this paper.

II. PMUs Based Power System State Monitoring

The deployment of PMUs has enabled many potential real-time applications that were not possible with the traditional measurement methods, such as state estimation, adaptive relaying and voltage instability enhancement. A typical IEEE 14 bus power system has been illustrated in Fig. 1. The communication networks connecting PMUs form a star topology, where all PMUs are supposed to transmit their measurements to the control center. When a PMU is installed at a bus in the power grid, it can provide the phasor voltage of that bus and all or part of the outgoing current phasor information on the branches connected to that bus. This is determined by the measuring terminal configuration of the PMUs. In this paper, it is assumed that each PMU will measure the bus voltage phasor and all connected branch current phasors.

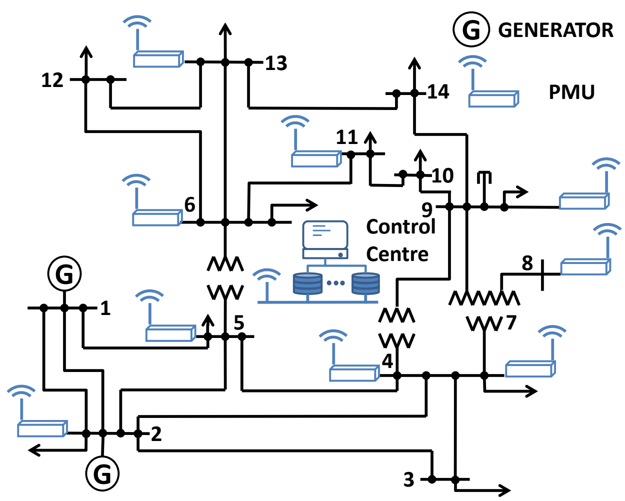

Fig. 1. IEEE 14 bus power system [8].

For an N bus power system, let the binary N by 1 column vector $\mathbf{X} = \{x_1, x_2, \ldots, x_N\}^T$ denote the PMU installation vector, whose elements are given by

$$x_i = \begin{cases} 1, & \text{if a PMU installed at bus } i, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

In addition, we assume that there are K PMUs in total, which are labelled as PMU_k, where $k = 1, 2, \ldots, K$. It is easy to verify that $K = \sum_{i=1}^{N} x_i$.

For a given power grid, it is assumed that the topology of the grid network is a prior. Then the elements of binary network connectivity matrix $\mathbf{H}$ can be given by

$$h_{ij} = \begin{cases} 1, & \text{if bus } i \text{ and } j \text{ are connected or } i = j, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

A bus will be observable if at least one PMU is placed at that bus or any bus incident to it [3]. Hence we can define the bus observability vector $\mathbf{b}$ as

$$\mathbf{b} = \mathbf{H}\mathbf{X}. \tag{3}$$

The element b_i in the bus observability vector $\mathbf{b}$ indicates the PMU numbers connected to or located at bus i. To maintain the entire power grid observable, it has to be guaranteed that every element $b_i \geq 1$, i.e. $\mathbf{b} \geq \mathbf{1}_N$.

III. Grid Observability Redundancy under Communication Constraints

In practical power systems, the measurements of PMU would vary with time. Hence the real-time grid status monitoring of the power grid has a stringent latency requirement. To maintain the real-time performance, each measurement from the PMUs will be valid within a determined time bound D_{max}. If the PMU measurements have been delayed longer than this maximum allowed latency, then these measurements may not be used, which results in a compromised power grid status monitoring performance. From the time a measurement is taken at the buses until it is used in the control center, almost every part will contribute to the overall latency. In this work, the communication latency has been mainly considered. In ideal cases, the communication systems should be designed to provide a 100 percent guarantee that the the communication delay d_k of PMU_k is smaller than the maximum allowed latency bound D_{max}. However, in practical systems, it has been identified that it is not feasible to provide a determine delay bound for the communication system in most fading channel environment [5]. Hence instead, we seek to provide a statistical probability $0 < p_k \leq 1$ to guarantee the communication delay within the maximum allowed latency bound D_{max}, namely

$$\Pr\{d_k \leq D_{\text{max}}\} \geq p_k. \tag{4}$$

For the real-time grid monitoring, if the latency of the measurements from a certain PMU exceeds the maximum valid time bound D_{max}, then this information will not be used. Hence the expected grid observability $\tilde{\mathbf{b}}$ under statistical latency guarantee can be given as follows

$$\tilde{\mathbf{b}} = \mathbf{H}\mathbf{\Lambda}_{\mathbf{P}}\mathbf{X}. \tag{5}$$

where $\mathbf{\Lambda}_{\mathbf{P}}$ denotes the N by N probability diagonal matrix, whose elements are defined by

$$\Lambda_i = \begin{cases} p_k, & \text{if PMU}_k \text{ installed at bus } i, \\ 1, & \text{otherwise.} \end{cases} \tag{6}$$

To keep the grid observable, it has to guarantee that $\tilde{\mathbf{b}} > \mathbf{1}_N$. The physical meaning of each element $\tilde{b}_i, i = 1, 2, \ldots, N$ of the expected grid observability $\tilde{\mathbf{b}}$ is that, the bus status information is available from an average of $\tilde{b}_i$ PMUs connected to the bus i. If any element $\tilde{b}_i$ is smaller than 1, then it means that the observability of this bus will not be guaranteed in a statistical view, and the power grid is vulnerable to the loss of the observability at that bus.

The grid observability is of great importance to the grid control or planning services. The loss of observability of bus status may result in serious consequences and should be avoided by the best efforts. Each PMU can provide the information of not only the bus it installed, but also the bus incident to it. Hence it is not necessary to have PMUs installed at every bus and it is still able to provide a desired degree of observability redundancy. Here we consider the overall grid observability redundancy r as the main interest metric, which is defined as

$$r = \mathbf{1}_N^T(\tilde{\mathbf{b}} - \mathbf{1}_N) \equiv \mathbf{1}_N^T \mathbf{H} \mathbf{\Lambda}_{\mathbf{P}} \mathbf{X} - N. \tag{7}$$

The metric r gives an evaluation of the overall power network observability redundancy. A larger r corresponds to the case that more PMUs are expected to be available to provide observability.

Providing higher grid observability redundancy under constraints has been one of the main interests for the power grid observability researches. By using (5), it is easy to verify that the grid observability redundancy r will be improved if the p_k for all PMUs are kept to be as close to 1 as possible. However, the parameter p_k is influenced by the bandwidth assigned to each PMU, whereas the total bandwidth B^{th} available to the whole system is bounded. This can be defined as a constraint for the bandwidth B_k assigned to each PMU$_k$, which is given by

$$\sum_{k=1}^{K} B_k \leq B^{\text{th}}. \tag{8}$$

In this paper, we focus on the problem of increasing the overall power system observability redundancy under the communication constraints, namely the maximization of r defined in (7), which can be formulated as

$$\max_{B_k, p_k} \quad \mathbf{1}_N^T \mathbf{H} \mathbf{\Lambda}_{\mathbf{P}} \mathbf{X} - N \tag{9}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} B_k \leq B^{\text{th}} \tag{10}$$

It can be noticed that there is a gap between the problem defined in (9) and the communication constraints (10), since it involves the optimization of both link layer and physical layer metrics in communication system, namely delays and bandwidths. To address this problem, a cross layer statistical delay analysis will be detailed in the next section.

IV. Cross Layer Statistical Delay Analysis

According to Shannon capacity theorem, the communication channel capacity is decided by the channel bandwidth and the signal-to-noise ratio (SNR). The variation of the instant SNR will affect the instant system throughput in the physical layer and result in latency at the link layer. In this paper, effective capacity theorem has been considered, which models the cross layer relation between the link layer queueing behaviour and the physical channel statistical characters [5].

Effective capacity is the dual concept of effective bandwidth, which is defined as the maximum constant rate that a fading channel can support under statistical delay constraints, which can be written as [5]

$$R(\theta, B) = -\frac{1}{\theta T} \ln \mathbb{E}\left\{e^{\theta T C}\right\}, \tag{11}$$

where C denotes the instant channel capacity during a single time block with a duration of T and the parameter θ is called QoS exponent, which is a non-negative value.

For PMU$_k$, when the transmitter sends uncorrelated circularly symmetric zero-mean complex Gaussian signals, (11) can be expressed as [9]

$$R_k(\theta_k, B_k) = -\frac{1}{\theta_k T} \ln \mathbb{E}_{\gamma_k}\{e^{-\theta_k T B_k \log_2(1+\rho_k \gamma_k)}\}, \tag{12}$$

where ρ_k is the average transmit SNR while γ_k is the instantaneous channel power gain.

If let d_k denotes the steady state delay at the buffer and D_{max} denotes the desired maximum delay, then the probability of d_k exceeding D_{max} can be given by [5]

$$\Pr\{d_k \leq D_{\text{max}}\} = 1 - e^{-\theta_k R_k(\theta_k, B_k) D_{\text{max}}}. \tag{13}$$

In this paper, we assume that the measurement from PMU$_k$ generates a constant source rate R_k^{th}. To avoid the system becomes unstable, the effective capacity of the wireless communication channel has to be no smaller than the source rate R_k^{th}, which is given by

$$R_k(\theta_k, B_k) \geq R_k^{\text{th}} \tag{14}$$

By exploiting effective capacity theory, the maximization problem of grid observability redundancy under communication constraints defined by (9) - (10) can be detailed as

$$\max_{B_k, p_k} \quad \mathbf{1}_N^T \mathbf{H} \mathbf{\Lambda}_{\mathbf{P}} \mathbf{X} \tag{15}$$

$$\text{s.t.} \quad R_k(\theta_k, B_k) \geq R_k^{\text{th}}, k = 1, 2, \ldots, K \tag{16}$$

$$p_k = 1 - e^{-\theta_k R_k(\theta_k, B_k) D_{\text{max}}}, k = 1, 2, \ldots, K \tag{17}$$

$$R_k(\theta_k, B_k) = -\frac{1}{\theta_k T} \ln \mathbb{E}_{\gamma_k}\{e^{-\theta_k T B_k \log_2(1+\rho_k \gamma_k)}\}, \tag{18}$$

$$\sum_{k=1}^{K} B_k \leq B^{\text{th}} \tag{19}$$

where (9) has been simplified to (15), which is due to the fact that the power grid bus number N is constant.

By allocating the communication resources using the optimal solutions from (15), the overall power grid observability redundancy performance will be maximized under constraint communication resources.

TABLE I

BUS OBSERVABILITY WITH A TOTAL OF 158KHZ BANDWIDTH.

Bus Number		1	2	3	4	5	6	7
Observability	Proposed Method	1.7364	2.6205	1.7045	4.3566	3.5296	3.4778	3.2256
	Default Method	1.0036	1.4947	0.4952	3.2912	2.4931	2.9963	2.2918
Bus Number		8	9	10	11	12	13	14
Observability	Proposed Method	1.4486	2.6202	1.7318	1.7480	1.7228	1.7228	1.7066
	Default Method	0.8202	2.2876	1.9744	1.9924	1.0026	1.0026	0.9846

TABLE II

PROBABILITY OF DELAY WITHIN MAXIMUM ALLOWED LATENCY WITH A TOTAL OF 158KHZ BANDWIDTH.

PMU bus location		2	4	5	6	7	8	9	11	13
Probability	Proposed Method	0.8204	0.8841	0.9161	0.9091	0.8432	0.6054	0.8929	0.8389	0.8137
	Default Method	0.0040	0.4912	0.9995	0.9983	0.8161	0.0042	0.9803	0.9941	0.0043

TABLE III

BUS OBSERVABILITY WITH A TOTAL OF 163KHZ BANDWIDTH.

Bus Number		1	2	3	4	5	6	7
Observability	Proposed Method	1.9895	2.9849	1.9882	4.9745	3.9813	3.9794	3.9692
	Default Method	1.5238	2.5013	1.5013	4.4936	3.5013	3.9332	3.7856
Bus Number		8	9	10	11	12	13	14
Observability	Proposed Method	1.9781	2.9850	1.9895	1.9901	1.9890	1.9890	1.9884
	Default Method	1.8086	2.9698	1.9992	1.9998	1.9334	1.9334	0.9328

TABLE IV

PROBABILITY OF DELAY WITHIN MAXIMUM ALLOWED LATENCY WITH A TOTAL OF 163KHZ BANDWIDTH.

PMU bus location		2	4	5	6	7	8	9	11	13
Probability	Proposed Method	0.9928	0.9954	0.9967	0.9964	0.9938	0.9843	0.9958	0.9937	0.9926
	Default Method	0.5238	0.9775	0.9999	0.9999	0.9929	0.8157	0.9994	0.9998	0.9334

V. CASE STUDY

In this paper, the case of IEEE 14 bus power system has been considered, which is shown in Fig. 1. Here we apply the primary and backup (P&B) method [4] for the PMU installation. According to P&B method, the PMUs are installed at bus 2,4,5,6,7,8,9,11 and 13, which provides the power grid with two independent PMU sets. Each of the primary and backup set can provide a full observation of the whole power grid. This redundancy allows the whole grid status observable even when multiple PMUs fail within only one set. Without loss of generality, it is assumed that all PMU generates measurement packages at the rate of 60kbps and the maximum allowed latency bound for these measurement packages is set to be 10ms.

Besides, the control center is assumed to be located at the center of the power grid. The average SNR ρ_k between control center and PMU$_k$ are related to their distances. In order to capture the effect of time varying fading effect, Rayleigh fading has been considered in this paper. The effective capacity under Rayleigh fading channels can be given by [7]

$$R_k(\theta_k, B_k) = -\frac{1}{\theta_k T} \ln {}_2F_0[\frac{\theta_k B_k T}{\ln 2}, 1, -\rho_k], \qquad (20)$$

where ${}_2F_0[\cdot]$ is the generalized hypergeometric function. The statistical probability of the PMU$_2$ communication delay within $D_{\max}$ has been given in Fig. 2. The Shannon capacity required for PMU$_2$ is 17.344kHz under the assumed scenario. It can be indicated from Fig. 2 that the latency bound will not be met with only minimum required bandwidth. In order to counteract the fading induced communication system fluctuation, extra bandwidths are needed for a desired performance, whose quantity can be obtained via (13).

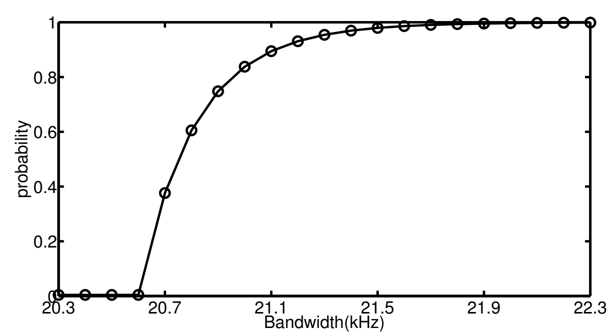

Fig. 2. The statistical probability of the PMU$_2$ communication delay within $D_{\max}$ under different bandwidths.

In order to prevent the failure of the communication systems formed by the PMUs, a redundancy bandwidth is always allocated to that transceiver nodes, which exceeds the real bandwidth that is actually required. As default bandwidth allocation method, the extra bandwidth beyond the minimum need of all PMU nodes will be evenly divided and allocated to each PMU. Whereas in our proposed method, the total bandwidth is allocated according to the optimal solution decided by (15). Since the problem defined in (15) is a nonlinear optimization problem, the active-set optimization algorithm is applied to find the best bandwidth allocation that maximize the grid observation redundancy.

The performances of individual bus observability as well as the statistical PMU latency guarantee probability have been given in Table I-IV, where the total available bandwidths are assumed to be 158kHz and 163kHz, respectively. To be specific, Table II and Table IV show that, under the same total bandwidth limit, the proposed method improves the overall PMU latency guarantee probability, which is at the cost of a relatively small probability drop of the PMU at bus 5, 6, 9 and 11. Also it can be told from Table I that, in the scenario where the total bandwidth is above 158kHz, the expected individual bus observability is all above 1 by using the proposed method. This means that the entire grid observability has been guaranteed from a statistical view point. However, under the same 158kHz bandwidth constraints, the default resource allocation method is very likely to lose the observability of bus 3, 8 and 14. It worth mentioning that, the minimum system required channel bandwidth is calculated to be 131.87kHz using Shannon capacity theory. By comparing the two scenarios where the total bandwidths are 158kHz and 163kHz, it also suggests that the proposed method makes better use of the redundant bandwidth for the observability performance improvement.

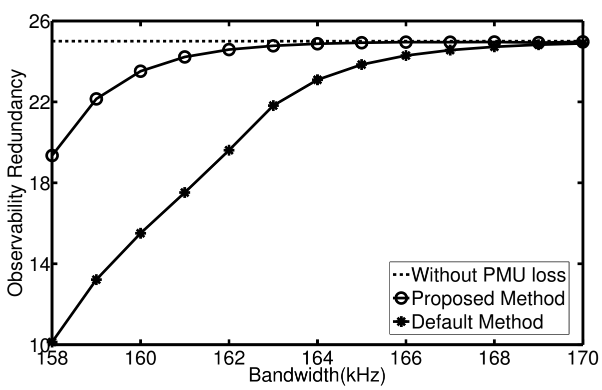

Fig. 3. The grid observability redundancy versus the bandwidth.

The overall grid observability redundancy under different total communication bandwidth scenarios have been illustrated in Fig. 3. Here we consider the PMU loss only results from the maximum latency bound violation. If without any PMU loss, the overall grid observability redundancy is 25 under the considered scenario. It can be seen from Fig. 3 that the proposed method provides a better grid observability redundancy performance across different bandwidths. With a total bandwidth of 163kHz, the proposed method can provide a close performance to the situation of no PMU loss, while it takes the default method to spend 169kHz to reach a similar performance. It should be noticed that this difference will be increased with more PMUs which are required for larger power grids.

VI. Conclusion

In this paper, a communication resource allocation method has been proposed, which is dedicated to improve the power grid observability performance. The proposed method aims at maximization of the overall power grid observability redundancy. The cross layer analysis method for the communication networks has been studied to address its influence on the power grid performance. A case study has been carried out on IEEE 14 bus power system, which has shown that the proposed method is able to help improve the power grid observability performance, specifically the overall power grid observability redundancy. This work has also investigated the aspect of providing statistical guarantees from the ICT system to improves the performances of power grid services. In the future work, we will further study the performances of various smart grid applications based on PMUs, such as state estimation and demand response.

ACKNOWLEDGMENTS

This work was supported by the UK EPSRC (grant no. EP/P005950/1), and the European Commissions Horizon 2020 framework programme (H2020/2014-2020) under grant agreement no. 734325 TESTBED project (http://testbed-rise.com/). The authors also acknowledge Dr Tilemachos Doukoglou and Dr Velissarios Gezerlis at OTE, Greece, for their useful discussions and comments during Mr M. You's OTE visit as a part of TESTBED project.

References

[1] T. Routtenberg, R. Concepcion, and L. Tong, "PMU-based detection of voltage imbalances with tolerance constraints," *IEEE Transactions on Power Delivery*, vol. 32, no. 1, pp. 484–494, Feb. 2017.

[2] K. G. Khajeh, E. Bashar, A. M. Rad, and G. B. Gharehpetian, "Integrated model considering effects of zero injection buses and conventional measurements on optimal PMU placement," *IEEE Transactions on Smart Grid*, vol. 8, no. 2, pp. 1006–1013, Mar. 2017.

[3] N. H. Abbasy and H. M. Ismail, "A unified approach for the optimal PMU location for power system state estimation," *IEEE Transactions on Power Systems*, vol. 24, no. 2, pp. 806–813, May 2009.

[4] X. Bei, Y. J. Yoon, and A. Abur, "Optimal placement and utilization of phasor measurements for state estimation," *PSERC Publication*, pp. 05–20, 2005.

[5] D. Wu and R. Negi, "Effective capacity: a wireless link model for support of quality of service," *IEEE Transactions on Wireless Communications*, vol. 2, no. 4, pp. 630–643, Jul. 2003.

[6] S. Agarwal, S. De, S. Kumar, and H. Gupta, "QoS-aware downlink cooperation for cell-edge and handoff users," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 6, pp. 2512–2527, Jun. 2015.

[7] M. You, H. Sun, J. Jiang, and J. Zhang, "Unified framework for the effective rate analysis of wireless communication systems over MISO fading channels," *IEEE Transactions on Communications*, vol. 65, no. 4, pp. 1775–1785, Apr. 2017.

[8] IEEE 14 bus power flow test case. [Online]. Available: https://www2.ee.washington.edu/research/pstca/pf14/pg_tca14bus.htm

[9] M. You, H. Sun, J. Jiang, and J. Zhang, "Effective rate analysis in weibull fading channels," *IEEE Wireless Communications Letters*, vol. 5, no. 4, pp. 340–343, Aug. 2016.