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The modelling of soil-tool interaction using the material point method

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ABSTRACT

The material point method (MPM) is a numerical tool able to model very large deformations in solid mechanics such as geotechnical problems. To date, soil-tool interaction modelling in the MPM is somewhat difficult, especially when the tool position is not stationary. Usually, the tool's boundaries do not coincide with the MPM grid and therefore boundary conditions cannot be imposed directly on the grid nodes. An implicit approach based on a penalty type method to impose essential boundary conditions in the MPM has been developed and used in the paper to demonstrate how to model frictionless soil-tool interaction mechanism [3]. This approach assumes that the tool is a rigid body as the deformation of the tool boundaries are insignificant. The same approach can also be used to model the interaction between structures and soil such as foundations.

Key Words: *material point method; essential boundary conditions; soil-tool modelling; offshore geotechnics*

1. Introduction

Since the development of the material point method (MPM) by Sulsky et al. [9] in 1994, it has been widely promoted as the next generation of numerical methods to model large deformations in materials such as soils. The method has two components: the material points which represent the volume of the solid, and the background grid that is used to solve the discretised linear system using same approach as the finite element method (FEM). While the MPM was developed in an explicit form [9], an implicit approach was later presented in [8] making it easy to transfer finite element analysis (FEA) technology to the MPM. After every load step, the material points' positions are updated and the background grid is reset to its original configuration. This means that the grid remains regular without any skewness, distortion and any associated numerical error.

The MPM has been successful in modelling large deformation problems such as landslides [1] and other geotechnical problems where boundary conditions are likely to coincide with the background grid nodes. When modelling soil-tool or soil-structure interaction in the MPM, it is often the case that the boundary of the tool or structure is arbitrary and does not coincide with the background grid nodes, so that it is not possible to impose these essential boundary conditions in the standard FEM approach. To overcome this problem, one can adopt the "moving mesh concept" [8], generally used when the boundary movement is known *a priori* and therefore the background grid nodes are moved simultaneously, but this unfortunately loses the advantageous regular grid concept of the MPM. The problem of imposing arbitrary essential boundary conditions on non-matching meshes, or so-called "embedded domains" (e.g. [5]), has been widely discussed with attempts to impose such boundary conditions (e.g. [4]) by using penalty, Lagrange multiplier or Nitsche methods. A simple penalty approach developed by Kumar *et al.* [7] to impose essential boundary conditions on a non-conforming grid FEA (called the implicit boundary method (IBM)), has been adopted in the MPM [2, 3] to impose both homogeneous and inhomogeneous Dirichlet boundary conditions. That means that arbitrary inclined fixed, rollers and non-zero displacement boundary conditions can be applied in the MPM to represent frictionless soil-tool interaction.

In this paper, we summarise the essential components for implementing the of IBM in the MPM, and demonstrate its effectiveness to model soil-tool interaction using an elastic domain pushed over a wedge-like tool.

2. Essential boundary conditions in the material point method

The IBM proposed for the structured grid FEM is based on a penalty approach proposed by Kantorovich and Krylov [6] during the 1950s, but only for orthogonal arbitrary positioned essential boundary conditions in the FEM[7]. The main concept of the IBM is to use Dirichlet step functions

$$d_\phi = \begin{cases} 0, & \phi < 0 \\ 1 - \left(1 - \frac{\phi}{\delta}\right)^2 & 0 \leq \phi \leq \delta \\ 1 & \phi > \delta \end{cases} \tag{1}$$

to identify the area inside/outside the solid region which are then used to activate/deactivate the standard approximation for displacement $\{u\}$ in the finite element sense and impose the essential boundary condition $\{u^a\}$. The displacement solution $\{u'\}$ is therefore obtained as follows,

$$\{u'\} = [D]\{u\} + \{u^a\}, \tag{2}$$

where $[D] = diag(d_x, d_y)$ in 2D. d_ϕ is set to unity to free the degrees of freedom in the ϕ-direction [3]. When Equation 2 is substituted in the standard FEM weak form, the strain-displacement matrix $[B]$ is now expanded and includes not only the finite element shape functions derivatives, but also the values of Dirichlet functions and their derivatives. Therefore the element stiffness matrix k^E with a volume V_e is expressed in four terms, i.e.

$$[k^E] = \int_{V_e} [B_1]^T[D^e][B_1]dV + \int_{V_e} [B_1]^T[D^e][B_2]dV + \int_{V_e} [B_2]^T[D^e][B_1]dV + \int_{V_e} [B_2]^T[D^e][B_2]dV, \tag{3}$$

where the first term is denoted as the standard finite element stiffness matrix $[K_1]$, while the second, third and fourth term of the implicit part are denoted as $[K_2]$, $[K_2]^T$ and $[K_3]$ and expressed as

$$[K_2] = \int_t [\bar{B}_1]^T \left(\int_0^\delta [\bar{D}_1]^T[D^e][\bar{D}_2][T]dn \right) [\bar{B}_2]dt, \tag{4}$$

$$[K_3] = \int_t [\bar{B}_2]^T \left(\int_0^\delta [T]^T[\bar{D}_2]^T[D^e][\bar{D}_2][T]dn \right) [\bar{B}_2]dt, \tag{5}$$

where $[\bar{B}_2]$ is a diagonal matrix contain the shape functions, $[\bar{B}_1]$ is identical to the finite element strain-displacement matrix $[B]$, $[\bar{D}_1]$ and $[\bar{D}_2]$ are expressed as

$$[\bar{D}_1] = \begin{bmatrix} D_1 & 0 & 0 & 0 \\ 0 & D_2 & 0 & 0 \\ 0 & 0 & D_1 & D_2 \end{bmatrix} \quad \text{and} \quad [\bar{D}_2] = \begin{bmatrix} \frac{\partial D_1}{\partial x} & 0 \\ 0 & \frac{\partial D_2}{\partial y} \\ \frac{\partial D_1}{\partial y} & \frac{\partial D_2}{\partial x} \end{bmatrix},$$

$[D^e]$ is the tangent stiffness matrix and $[T]$ is a rotation matrix in the Cartesian coordinate system used to transform the inner integrals of $[K_2]$ and $[K_3]$ by a rotation angle θ to achieve an inclined boundary condition as shown in Figure 1. Non-zero prescribed displacement boundaries are imposed by calculating the equivalent reaction force as

$$\{f_R^E\} = [[K_2] + [K_2]^T + [K_3]]\{u_a\} \tag{6}$$

caused by a nodal displacement $\{u_a\}$, and are included in the linear system to be solved.

Another important step to impose arbitrary essential boundary conditions in the MPM is the discretization of the continuum into a set of material points. In the initial pre-processing step, the background grid is usually populated with a set of material points with associated domains and volumes. If the material point domain lies complete inside the solid region, this point is retained, while if the material point domain lies completely outside the solid region, this point is removed. If a material point domain lies on the boundary of the problem, the material point domain is truncated to reflect the new solid region only, and the volume is adjusted accordingly. In the MPM, the material points are used similar to Gauss points in the standard FEM to evaluate the stiffness matrix $[K_1]$, and therefore grid elements are omitted from the system of equations if they containing material points with a tiny fraction of volume measured to a given tolerance. The implementation of the IBM in the MPM has now been successfully benchmarked [2, 3], and is used in the next section on a soil-tool interaction problem.

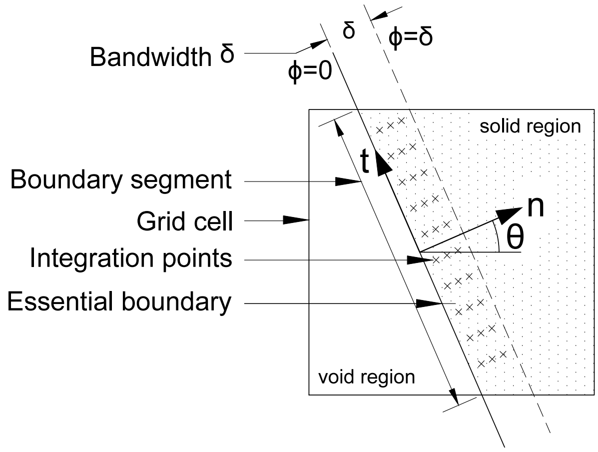

Figure 1: Essential boundary through a grid element

3. Penetration of a wedged-like tool

In this section, a wedge-like tool penetrating an elastic domain is used to demonstrate how soil-tool interaction can be modelled by using the IBM in the MPM. The problem consists of a solid elastic continuum confined between two horizontal and an inclined roller boundary conditions, while the elastic domain (shaded area) is being pushed toward the wedge as shown in Figure 2 (a). The stationary inclined boundary is used to represent a frictionless wedge onto which the elastic domain will be compressed while moving horizontally to the left. The problem is solved on 2mm x 2mm background grid and a single material point in every grid element is used, as shown in Figure 2 (b). The elastic body has a Young's Modulus of 1000 MPa and a Poisson's ratio of 0.3. The bandwidth δ was set to $\delta = 5 \times 10^{-6}\ h$, where h is the background grid size. An incremental displacement of 6mm per load step was used to push the elastic domain toward the wedge. The deformations of the body at the 1^{st}, 3^{rd}, 5^{th} and 6^{th} load steps are shown in Figures 2 (c-f) respectively, where the contour plots represent the x-displacement of the material points. Despite large load steps, that the material points clearly move along the wedge without crossing the boundary, causing the elastic domain to "funnel" through the space between the wedge and top horizontal roller boundary conditions. Such wedge type problem can be easily used to represent a frictionless soil-tool interaction such as ploughing schemes and other similar problems.

4. Conclusions

In this paper we have described how an implicit boundary method has been used to model roller and other boundaries in the MPM. This allows the modelling of frictionless soil-tool interaction problems. In future work, the IBM will be developed to account for friction when this is important to the overall mechanism of the problem.

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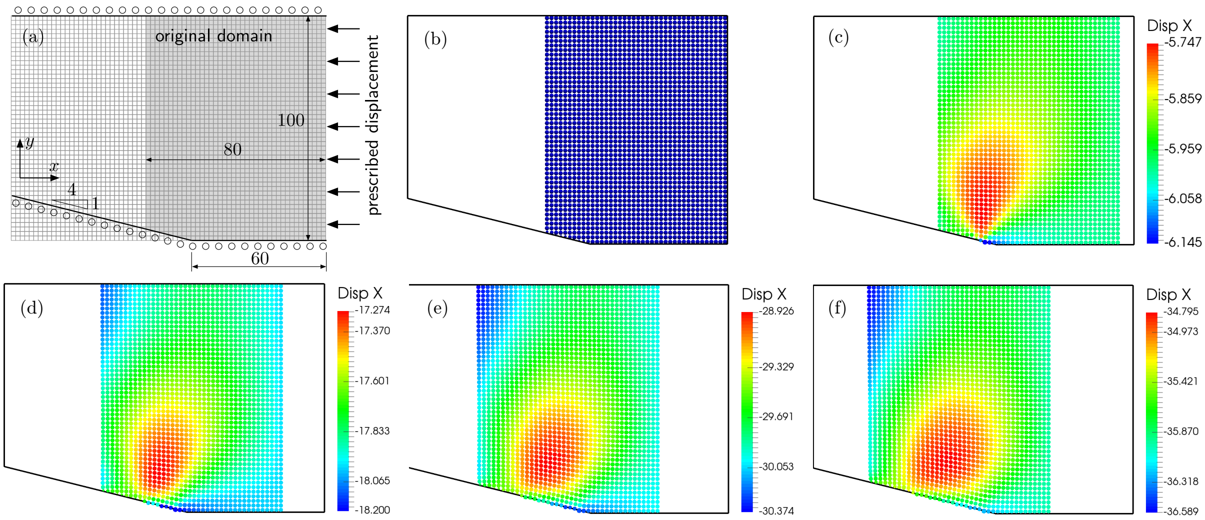

Figure 2: Rigid body penetration into an elastic domain: (a) problem dimensions, (b) initial MP discretisation and (c) through (h) deformed material point positions coloured according to their x-displacement.

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