Multimodal Lamb Wave Identification Using Combination of Instantaneous Frequency with EMD

Zhe Wang
Department of Electrical Engineering
Tsinghua University
Beijing, China
z-w16@mails.tsinghua.edu.cn

Songling Huang
Department of Electrical Engineering
Tsinghua University
Beijing, China
huangsling@mail.tsinghua.edu.cn

Hongyu Sun
Department of Electrical Engineering
Tsinghua University
Beijing, China
shymanuscript@163.com

Shen Wang
Department of Electrical Engineering
Tsinghua University
Beijing, China
wangshen@mail.tsinghua.edu.cn

Qing Wang
Department of Engineering
Durham University
Durham, UK
qing.wang@durham.ac.uk

Wei Zhao
Department of Electrical Engineering
Tsinghua University
Beijing, China
zhaowei@mail.tsinghua.edu.cn

***Abstract*—Lamb wave is guided plate wave that has potential in industrial inspection for non-destructive evaluation. However, the multimodal nature of Lamb wave obstructs the extraction and interpretation of information in signal. This work proposes a Lamb wave mode identification method combining instantaneous frequency (IF) with empirical mode decomposition (EMD). Firstly, EMD is implemented on multimodal signal and separates it into individual mode functions. Then the individual functions are analyzed by high-resolution time-frequency distribution. Finally, IF curve is obtained in the time-frequency domain and corresponding time information is used to calculate wave velocity. Lamb wave mode is determined by comparing the wave velocity with theoretical value. To verify the effectiveness of the proposed method, simulation presents the situation of boundary reflection and experiment presents the mode conversion by defect. Both simulation and experiment demonstrate the good performance of the mode identification method.**

***Index Terms*—Lamb wave, instantaneous frequency, mode identification, time of flight**

I. Introduction

Metal components used in industry often face the long-standing problem brought by defect. Guided wave inspection technology provides an efficient non-destructive testing method [1]. Lamb wave propagating through the metallic plate can give valuable information about the health status of the structure [2].

Time of flight (TOF) of Lamb wave is the commonly used information to determine the location, even the shape of defect [3]. However, the multimodal nature of Lamb wave influences the precise measurements of TOF. Several time-frequency analysis methods have been developed to study multiple mode Lamb wave signal. Hilbert transform and Hilbert-Huang transform had been used as a postprocessing tool to evaluate the instantaneous characteristics of Lamb wave [4]. The ridge tracking and Vold-Kalman filter were introduced to separate interfered wave modes [5]. The empirical mode decomposition (EMD) based short time Fourier transform to extract TOF was proposed to recognize overlapped Lamb wave [6]. The squeezed wavelet transform was applied to analyze the simulated broadband Lamb wave signal and the advantage of this transform was the ability of signal reconstruction [7].

Instantaneous frequency (IF) was originally defined for frequency modulation in communications. Then the concept of IF was expanded to non-stationary signal. Ville incorporated early work and related IF to the derivative of signal phase [8]. A unified approach to time-frequency representations (TFR) and IF was analyzed afterwards [9]. Wigner distribution (WD) with adaptive window width was applied to estimate the IF [10]. For signal with non-linear IF, L-Wigner distribution was proposed [11]. B-distribution, a high-resolution quadratic distribution was also designed to reduce cross-terms in TFR for multicomponent signals [12]. An approach based on ant colony optimization and TFR was developed to analyze IF of non-stationary signals embedded in high noise [13].

In order to distinguish different Lamb wave modes, this paper proposes a mode identification method combining IF with EMD. Lamb wave signal is decomposed into individual functions in time domain and then IF curve is extracted in time-frequency domain to determine the wave mode. Both simulation and experiment are performed to verify the effectiveness of the proposed method.

The research is organized as follows. Section II presents the theoretical analysis of Lamb wave and the multimodal characteristic is discussed. In section III, mode identification method combining IF with EMD is proposed and its process

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is summarized in detail. The validation of mode identification method is conducted in both simulation and experiment in Section IV. Section V gives concluding remarks.

II. Theoretical Analysis of Lamb Wave

Lamb wave is one type of elastic waves existing in thin plate. When Lamb wave propagates along the plate structure, the vibration covers the whole plate through the thickness. The displacements of Lamb wave have two components which are shown in Fig. 1.

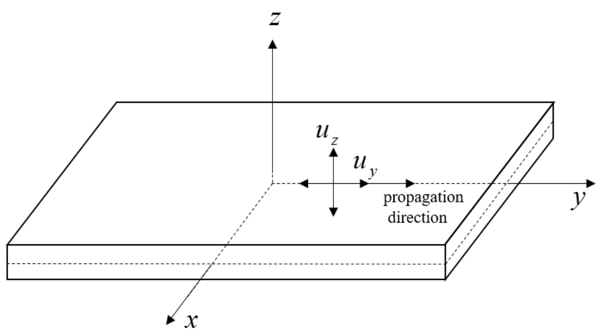


Fig. 1. Displacement components of Lamb wave.

According to the different forms of vibration, Lamb wave is decoupled into symmetric mode (S mode) and antisymmetric mode (A mode). Based on linear elasticity theory, Lamb wave can be solved and expressed by Rayleigh-Lamb equation as follows:

$$S\ mode: \quad \frac{\tan(qh)}{\tan(ph)} = -\frac{4k^2pq}{(q^2-k^2)^2} \tag{1}$$

$$A\ mode: \quad \frac{\tan(qh)}{\tan(ph)} = -\frac{(q^2-k^2)^2}{4k^2pq} \tag{2}$$

where, $h = \frac{1}{2}d$, $p^2 = \frac{\omega^2}{c_L^2} - k^2$, $q^2 = \frac{\omega^2}{c_T^2} - k^2$, $k = \frac{\omega}{c_p}$. d is the plate thickness, c_L is the longitudinal wave velocity. c_T is the transverse wave velocity, k is the wave number and c_p is the phase velocity. Therefore, the velocity of Lamb wave depends on the parameters of plate and excitation frequency.

The group velocity c_g, which is the velocity of Lamb wave packet, can be derived from phase velocity as $c_g = \frac{d\omega}{dk}$. The Rayleigh-Lamb equation can only be solved numerically. And then the group velocity can be obtained and shown in Fig. 2, which is also called the dispersion curve. The velocity is considered as the function of frequency-thickness product.

From Fig. 2, it is clear that many possible Lamb wave modes may exist in the same frequency-thickness product. And mode interference is unavoidable when applying Lamb wave. In the case discussed in following part, the two fundamental modes A_0 and S_0 mode are generated and analyzed.

III. Method of Mode Identification

A. Instantaneous Frequency

Lamb wave signal is typically non-stationary. The signal spectrum varies as a function of time. To describe and analyze the non-stationary signal, IF provides a potential tool.

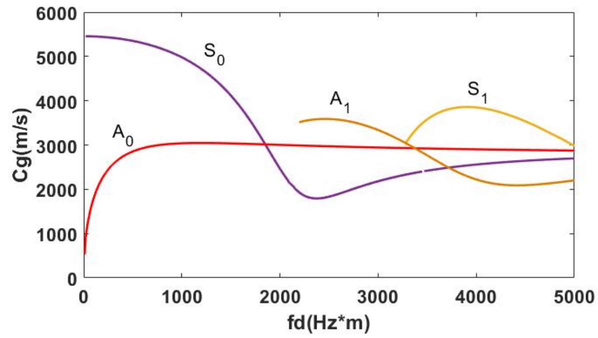


Fig. 2. Multimodal characteristics of Lamb wave.

The IF of a signal $x(t)$ can be defined as the derivative of the phase of its analytic signal $z(t)$ [8]:

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\arg(z(t)) = \frac{1}{2\pi}\frac{d\varphi(t)}{dt}. \tag{3}$$

If $x(t)$ is complex-valued, it can be expressed as $a(t)e^{j\varphi(t)}$. $a(t)$ is known as the instantaneous amplitude and $e^{j\varphi(t)}$ is known as the instantaneous phase. If $x(t)$ is real-valued, it can be converted to its corresponding analytic form by Hilbert Transform. The Hilbert Transform of $x(t)$ is

$$\hat{x} = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau. \tag{4}$$

Then the analytic signal is

$$z(t) = x(t) + j\hat{x}(t) = a(t)e^{j\varphi(t)}. \tag{5}$$

According to (3), the IF of complex-valued and real-valued signal can be obtained.

IF can also be defined using the first moment of time-frequency distribution (TFD). IF is the weighted average of the frequencies at time t:

$$f_i(t) = \frac{\int_{-\infty}^{+\infty} f\rho(t,f)df}{\int_{-\infty}^{+\infty} \rho(t,f)df} \tag{6}$$

where, $\rho(t,f)$ is the TFD. For signal with linear frequency, WD gives the best energy concentration in time-frequency representation. However, when the signal contains multiple components, WD brings serious cross-terms which decreases the time-frequency resolution.

To reduce the interference of cross-terms, the general quadratic TFD is considerd. The general form can be introduced as

$$\rho(t,f;g) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} z(u+\frac{\tau}{2})z^*(u-\frac{\tau}{2}) g(\theta,\tau)e^{-j2\pi(\theta t+f\tau-u\theta)}dud\tau d\theta \tag{7}$$

where $z(t)$ is the analytic signal, u is variable of integration, τ is the time-shift or lag, θ is the frequency-shift or Doppler and $g(\theta,\tau)$ is the kernel function or parameterization function. If $g(\theta,\tau) = 1$, then the distribution becomes WD. $g(\theta,\tau)$, which performs as a 2-D filter, determines the properties of the distribution. Therefore, choosing suitable kernel is significant to reduce the cross-terms. By designing the kernel, B-distribution is proposed to enhance the time-frequency resolution [12]. Its kernel in time-lag domain is given by

$$G(t,\tau) = \left[\frac{|\tau|}{\cosh^2(t)}\right]^{\alpha}. \tag{8}$$

In the kernel, $1/\cosh^2(t)$ makes it has the narrowest main-lobe and α is an adjustable real number. $g(\theta,\tau)$ is the Fourier transform of $G(t,\tau)$. α influences the sharpness of cutoff of $g(\theta,\tau)$. And α should be from 0 to 1. With the kernel, B-distribution satifies (6). Therefore, the first moment of B-distribution produces IF.

B. Empirical Mode Decomposition

Lamb wave has the nature of multiple modes which can be divided into A and S mode. When Lamb wave encounters with defect, the acoustic wave will scatter. Besides, the phenomenon of mode conversion will occur.

Hilbert-Huang Transform is proposed to analyze non-stationary and non-linear signal [14]. It contains two processes, including the EMD and Hilbert spectral analysis.

EMD, also called sifting process, is essentially a process to make signal approximately stationary. EMD decomposes the signal into intrinsic mode function (IMF) components. For signal $x(t)$, firstly find all local maxima and minima. The maxima and minima are connected using cubic spline line, respectively, and then form the upper envelope $u(t)$ and lower envelope $l(t)$. The mean of the two envelopes is:

$$m(t) = \frac{u(t)+l(t)}{2}. \tag{9}$$

The first IMF can be estimated as:

$$h_1(t) = x(t) - m_1(t). \tag{10}$$

To eliminate the riding waves, the sifting procedures should be performed repeatedly. Assuming $h_1(t)$ as the signal, obtain the upper envelope $u_{11}(t)$ and lower envelope $l_{11}(t)$. Then the mean curve $m_{11}(t)$ can be calculted and the next estimation of IMF is:

$$h_{11}(t) = h_1(t) - m_{11}(t). \tag{11}$$

The stopping critera for this iteration can be calculated from two sifting results:

$$SD_k = \sum \frac{(h_{1(k-1)}(t) - h_{1k}(t))^2}{h_{1(k-1)}(t) * h_{1(k-1)}(t)}. \tag{12}$$

The reference value is typically 0.2-0.3. Once SD_k is lower than the reference value, the process can be terminated and the first IMF is determined:

$$c_1(t) = h_{1k}(t). \tag{13}$$

Then the residual signal is

$$r_1(t) = x(t) - c_1(t). \tag{14}$$

To separate the residual IMF, $r_1(t)$ is considered as the original signal and repeat the aforementioned procedures. Finally, $x(t)$ is decomposed into several IMFs and the last residual signal:

$$x(t) = \sum_{k=1}^{m} c_k(t) + r_m(t). \tag{15}$$

C. Combination of IF with EMD

Based on the analysis of frequency at specific moment and mode decomposition, the method for multimodal Lamb wave identification is proposed. For signal containing multiple modes, it is firstly decomposed into several IMF components by EMD. Then the individual components are processed to obtain its mode. The B-distribution, one of the reduced interference distribution, is implemented on the components. By utilizing the relation of IF and TFD, the IF is calculated and corresponding time information is adopted to identity the mode.

The procedures of the mode identification method are summarized in the following steps:

a) Choose suitable excitation toneburst to generate the Lamb wave and receive the signal $x(t)$ to be analyzed.

b) Implement EMD on $x(t)$ into its IMFs using equations from (9) to (15) and choose concerned IMFs with potential Lamb wave modes.

c) Implement B-distribution on IMFs using equations from (7) to (8) and obtain corresponding TFD $\rho(t,f)$.

d) Calculate IF by (6). IF can be considered as a function of time and presented as one curve in the time-frequency plane.

e) According to the center frequency of toneburst, extract the corresponding time information in IF curve.

f) Calculate the velocity of IMF components and compare it with the Lamb wave dispersion curve to determine the mode.

IV. Results and Discussion

To validate the effectiveness of the proposed mode identification method, corresponding simulations and experiments have been conducted.

A. Simulation Results

The simulation of Lamb wave is performed using finite element method by Comsol Multiphysics software. In the finite element model, the plate is made of steel and its height is 4 mm. The center frequency and cycle number of excitation signal is 250 kHz and 10, respectively. From the dispersion curve in Fig. 2, two fundamental Lamb wave modes of A_0 and S_0 will be generated. In the process of simulation, the total displacement captured at 100 μs is displayed in Fig. 3.

From Fig. 3, two modes appear in the same moment with a space distance. Considering the velocities of A_0 and S_0 mode are 3167.21 m/s and 5143.86 m/s respectively, the generated space distance brought by velocity difference is 197.7 mm. The simulation result is in accord with the theoretical calculation.

The receiver of Lamb wave is set between the excitation point and plate boundary. The receiver is 3050 mm away from the excitation point and 950 mm away from the boundary. The

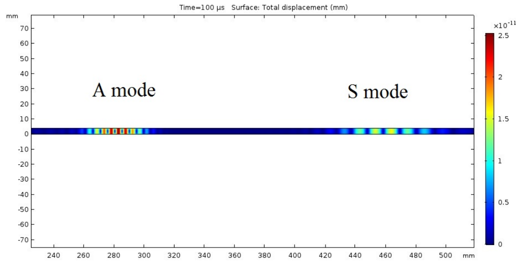
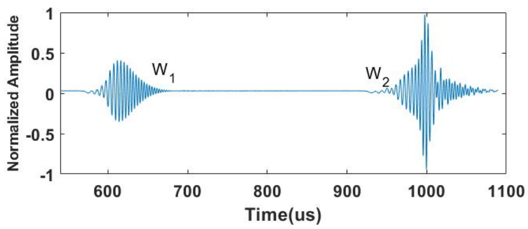
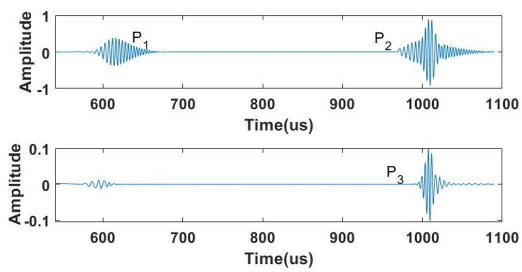


Fig. 3. The total displacement captured at 100 μs.

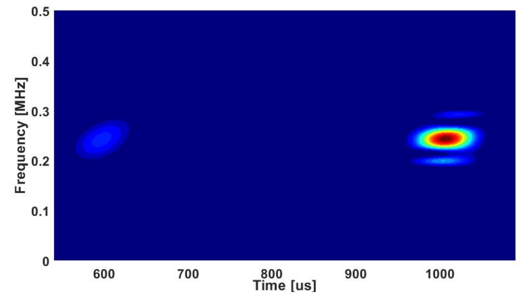


(a) Original waveform

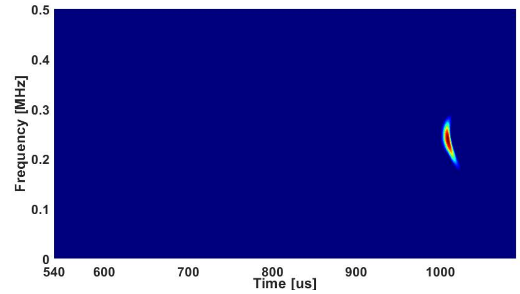


(b) The first IMF and second IMF

Fig. 4. IMF of simulated waveform by EMD.



(a) TFR of the first IMF



(b) TFR of the second IMF

Fig. 5. TFR of the separated IMFs.

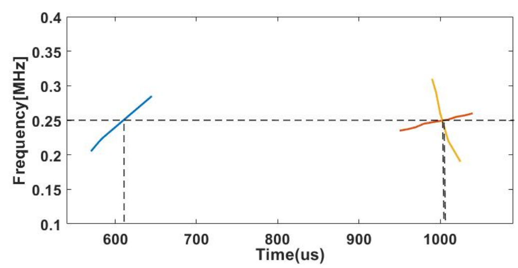


Fig. 6. IF curves extracted from TFD.

received waveform is shown in Fig. 4(a). Implement EMD on the wavepackets and obtain IMFs, which are presented in Fig. 4(b).

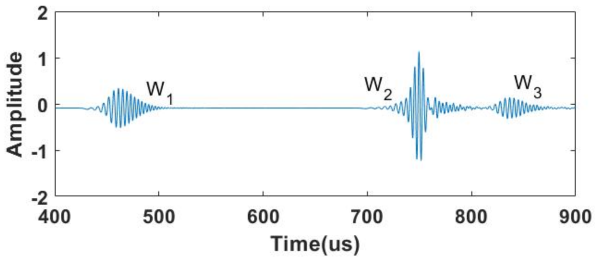
From Fig. 4(a), two distinct wavepackets exist in the original waveform. By mode decomposition on each wavepackets, the first IMF and second IMF are shown in Fig. 4(b). The third IMF is ignored because its amplitude is too small. It is clear that the second wavepacket W_2 is separated into two parts: P_2 and P_3. One part is the direct wave and the other is the reflected wave. To identify the wave mode, the TOF is extracted by applying the proposed method. Firstly, B-distribution is conducted and the obtained TFR is shown in Fig. 5. It indicates that the cross-terms are removed in B-distribution for IMF1. Secondly, the first moment of B-distribution is calculated to obtain IF. IF forms curves in the time-frequency domain, shown in Fig. 6. Lastly, the corresponding time of center frequency 250 kHz is read from the IF curve. By subtracting the excitation time of 20 μs, TOF is obtained.

The TOFs of the three wavepackets (P_1, P_2 and P_3) are 593.5, 966.3 and 963.4 μs, respectively. In view of the the propagation distance, the corresponding velocity is 5139.0, 3156.4 and 5138.1 m/s. Therefore, by comparing the velocities with those in the dispersion curve in Fig. 2, the wavepackets of $P1$, $P3$ are S_0 mode and $P2$ is A_0 mode.

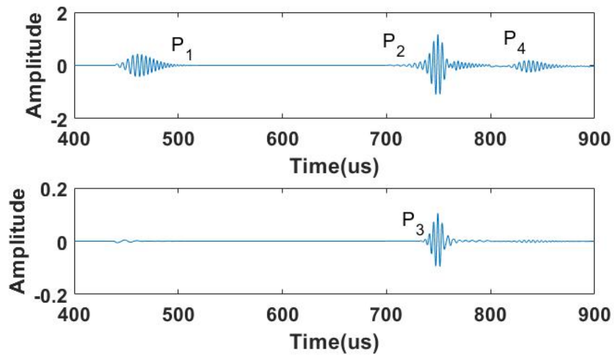
B. Experiment Results

The Lamb wave experiment system is established on steel plate and corresponding experiments have been conducted. The plate contains an artificial defect of 20 mm $\times$ 10 mm $\times$ 3mm. In the experiment system, electromagnetic acoustic transducers (EMATs) are used as acoustic transmitter and receiver. The received Lamb wave signal is amplified and filtered by the signal conditioning circuit. Then the signal is sampled and sent to the computer for further analysis.

The receiver of Lamb wave is placed between the transmitter and defect. The receiver is 2280 mm away from the transmitter and 720 mm away from the defect. The received waveform and its EMD are shown in Fig. 7(a). The second wavepacket W_2 in the original signal is decomposed into two wavepackets: P_2 and P_3. Compare the IMF1 with that in simulation result in IV-A, $P4$ following $P2$ is additional wavepacket which might



(a) Original waveform



(b) The first IMF and second IMF

Fig. 7. IMF of experimental waveform by EMD.

be introduced by mode conversion because of the interaction of Lamb wave with defect. To determine the modes of the four wavepackets, B-distribution is implemented and IF curve is calculated.

TABLE I
TOF AND MODE OF THE FOUR WAVEPACKETS.

Wavepackets	**TOF** (μs)	**Distance**(mm)	**Velocity**(m/s)	**Mode**
P1	445.1	2280	5122.4	S_0
P2	723.6	2280	3150.9	A_0
P3	726.8	3720	5118.3	S_0
P4	224.4	720	3209.0	A_0

The propagation time in terms of the center frequency in different wavepackets can be obtained easily from the IF curve. Comparing the time with the excitation time of 20 μs, TOFs and velocities are calculted and shown in Table. I. According to the dispersion curve, the wave modes are determined. $P1$ and $P3$ are S_0 mode. The other two wavepackets $P2$ and $P4$ are A_0 mode.

V. Conclusion

In the work described here, the method using combination of IF with EMD is proposed for multimodal Lamb wave identification. The non-stationary Lamb wave signal is decomposed into individual functions by EMD. Then a high-solution time-frequency representation is applied to analyze the functions and IF curves are obtained. Through the extraction of TOFs and the calculation of velocities, the modes are determined according to the dispersion curve. The simulation and experiment demonstrate the effectiveness of the proposed method. Compared with other mode identification methods, the combination of IF with EMD not only distinguishes different Lamb wave modes, but also provides precise measurement of TOF for further defect locating and imaging.

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