Quality perception and discrimination thresholds in quantised triangle meshes

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ABSTRACT

At certain stages of the graphics pipeline, and most notably during compression for transmission and storage, triangle meshes may undergo a fixed-point arithmetic quantisation of their vertex coordinates. This paper presents the results of a psychophysical experiment, where discrimination thresholds between the original unquantised triangle meshes, and the quantised at various levels of quantisation versions of them, were estimated. The experiment had a two-alternative forced choice design. Our results show that the amount of geometric information of a mesh, as measured by its filesize after compression, correlates with the discrimination threshold. On the other hand, we did not find any correlation between the discrimination thresholds and the quality of the underlying meshing, as measured by the mean aspect ration of the mesh triangles.

Keywords: Triangle meshes, quantisation artifacts, two-alternative forced choice

1. INTRODUCTION

Triangle meshes have emerged as the ubiquitous standard for 3D content representation for most graphics applications. Being, essentially, piece-wise linear representations of surfaces, triangle meshes are simple and scalable, and benefit from the existence of specialised sophisticated algorithms covering the whole graphics pipeline, from mesh generation, to processing and rendering, to transmission and storage.

The vertex coordinates of a triangle mesh are usually represented by 32-bit floats. However, at various stages of its life cycle, and most notably during compression for transmission or storage, the vertex coordinates may be quantised and represented in a fixed-point arithmetic, typically, by 12, 16 or 24 bits per vertex coordinate. This paper presents the results of a psycho-physical experiment, part of a series of similar experiments conducted by the authors,[1,2] aiming at studying the visual effect of such quantisations. In particular, given that vertex coordinate quantisation is an irreversible process, we study the discrimination threshold, above which an observer is not able to perceive quantisation artifacts.

In,[1] the main focus of the experiment was on the impact of the quantisation method. We compared rounding, where the least significant bits are put to zero, and dithering where the least significant bits are randomised, and found that, in general, dithering has a slightly higher discrimination threshold. In,[2] the focus of the experiment was on the impact of the number of triangles in the mesh, and that of the rendering method. It was found that, generally, larger meshes have higher discrimination thresholds, and also that renderings with a higher specular component have higher discrimination thresholds.

This paper studies the impact of the geometry of the mesh, that is, how the shape of the 3D model and the properties of the underlying mesh are related to the discrimination threshold. Our study is based on a two Alternative Forced Choice, psycho-physical experiment, where two stimuli of one model are presented — the original and one which is quantised at a certain level — and the participant chooses the one with the higher visual quality. Four different 3D models were used, the Max-Planck, the Cone, the Sphere, and the Human-Head, fixing all the experimental parameters that had been studied previously. In particular, the quantisation

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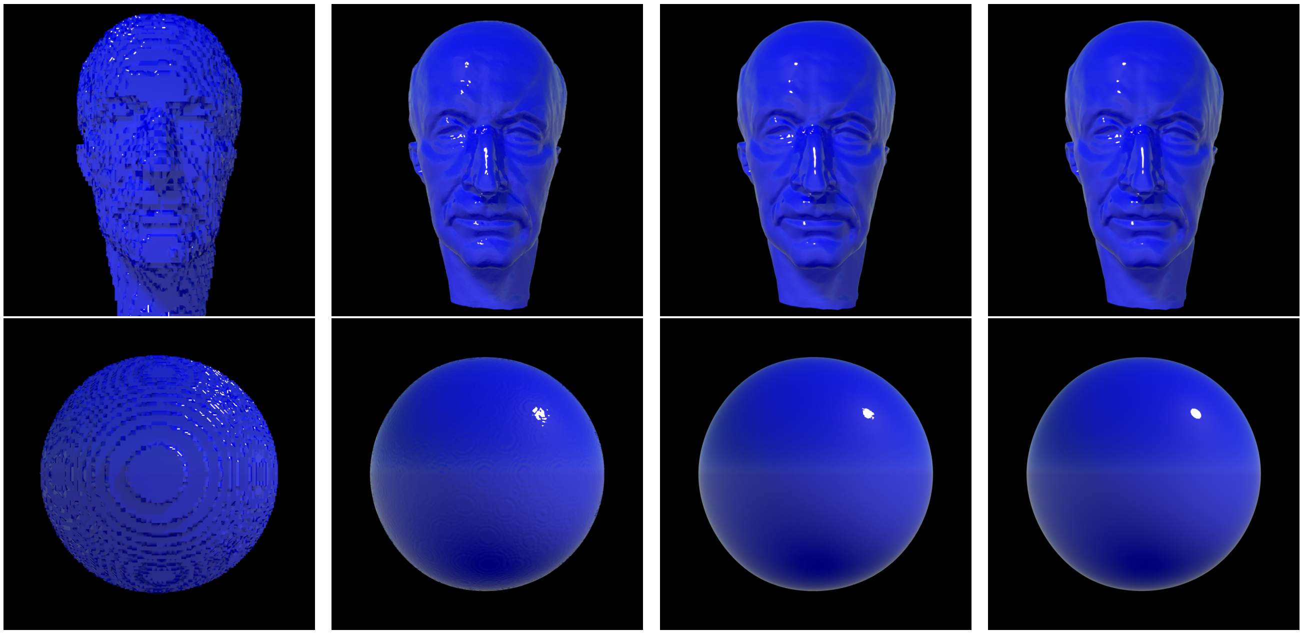

Figure 1. From left to right: The Max-Planck model (top) and the Sphere model (bottom) quantised at 8,12 and 14 bits per vertex coordinate, and the original unquantised model.

method was rounding, all meshes consisted of a large number of triangles, between 200K and 315K, and the single rendering method that was used in all cases, had a high specular component.

The first hypothesis was that there is an inverse correlation between the amount of geometric information carried by a 3D model and its discrimination threshold. In particular, it was hypothesised that the simpler in shape synthetic models, the Sphere and the Cone, will have higher discrimination thresholds, because it will be easier for a participant to detect quantisation artifacts on them, while, in contrast, quantisation artifacts will be more difficult to detect on the more complex surfaces of the natural models. This hypothesis was verified and moreover, the post-hoc analysis found a strong, statistically significant correlation between the discrimination threshold and the filesize of the compressed mesh, which was used as measure of the amount of geometric information carried by the mesh.

For an illustration of our hypothesis, Figure 1 shows, at various levels of quantisation, the Max-Planck model, which has the largest filesize when compressed, and the Sphere models which has the smallest compressed filesize. Notice that at 12 bits per vertex coordinate, quantisation artifacts in the form of surface texture are still visible on the Sphere, while one cannot detect this type of artifacts on the Max-Planck model.

The second hypothesis was that the average quality of the triangles of the mesh has an impact on the discrimination threshold. In particular it was hypothesised that meshes comprising long thin triangles will have higher discrimination thresholds compared to meshes comprising well-rounded, near equilateral triangles. That was also a quite intuitive hypothesis, given that the same amount of spatial perturbation will most likely cause a larger perturbation of the normals of the thin triangles, which in turn will be easier to detect in a rendering of high specularity.[3] The measure of the thinness of a triangle was the aspect ratio, that is, the ratio of the smallest edge-length by the largest. Our experiment did not verify that second hypothesis, as there was no significant correlation between the mean aspect ratio of the mesh triangles and the discrimination thresholds.

1.1 Background

Quantise the triangle mesh is the first step of all mesh compression algorithms.[4] In order to encode the vertex coordinate, most of application frequently utilise the 32-bit floats. On the other hand, when a triangle mesh is needed in the compressed form, the use of 16 bits per vertex coordinate seems to be the usual practice in mesh compression.[4]

Quantisation techniques are mainly studied from the perspective of signal theory.[5] According to an extensive survey of the technique in,[6] rounding is the most commonly used and traditional example of quantisation for density estimations via histograms back in 1897.[7] Here, the spatial coordinates of the mesh vertices were quantised by rounding. On the other hand, quantisation techniques applied to the various frequency domain representations of the mesh geometry, see for example,[8] might have significant theoretical interest, as well as significant applications, but are nevertheless less relevant to the everyday real-life use of meshes.

A previous study by the authors for determining the visual effect of quantisation, used a simple yes/no task experiment and was aimed at determining a discrimination threshold beyond which the quantized mesh is not perceived to differ from the original.[1] However, the focus there was on understanding the effect the choice of quantisation method has on the threshold, focusing in particular on the effect of dither.

The main alternative methodological approach to the subjective psycho-physical experiment presented in this paper, would have been the use of an objective mesh quality metric. Various such metrics have been proposed in the literature, measuring mesh quality based on criteria, such as size, shape, and smoothness,,[9],[10],[11],[12].[13] There are also various mesh quality metrics which are computed as averages over the whole mesh of a single triangle quality metric. Examples of such metrics, which are often used in practice for mesh optimization, include: edge length ratio,[14] area,[15] edge length root mean square,[14] inverse mean ratio,[16] and aspect ratio.[17]

We note that the use of objective metrics, such as those mentioned above, seems to be a more appropriate methodology in the context of CAD and Finite Element Method applications, while in the context of visual applications they are mostly employed as a cheap alternative to the systematic user studies. For example, as Vanhoey et al.[18] stated, for two main reasons, only a few subjective studies have been performed in the field of interactive visualisation: firstly, it is a relatively new field with less than twenty years history; secondly, perceptual experiments are expensive and time-consuming processes. In some of the first studies based on psychophysical experiments, Rushmeier et al.[19] study the effect of geometry and texture resolution on the perceived quality, however, all their models are unquantised and the geometric resolution of the model is its number is triangles. In Rogowitz and Rushmeier,[20] a psychophysical experiment is conducted to establish perceptual differences between animated 3D models and 2D still renderings of them. Away from triangle meshes,[21] conduct a psychophysical experiment to study the effect of the wireframe and the texture resolution on the perceived visual quality of wireframe models.

The experiment presented in this paper is based on a two-alternative forced-choice design. The simplicity of the method makes for experiments that are relatively simple to design and run, and fare favourably compared to other, more complex designs, especially when the number of participants is small.[22] In our context, the main benefit from the simplicity of the experimental design is that it minimises the risks to the validity of the results from any misunderstandings, or subjective interpretations of the tasks by the non-expert participants. Indeed, when conducting analogous experiments based on the more complex *Maximum Likelihood Difference Scaling* design,,[23],[24] we found that although specific users would return meaningful results, the statistical aggregation of all users was not possible because of the large proportion of participants who either did to understand the task, or interpreted it in their own subjective way. Another possibility regarding the type of the psychophysical experiment would have been the use of a task based experiment, as for example in,[25] where the users are asked to recognise, as fast as possible, 3D models presented to them at various resolutions. We note that such designs are rarely used in the assessment of perceptual quality of 3D models, as they cannot detect very fine grain differences and, moreover, they suffer from high variance between participants.

2. EXPERIMENTAL DESIGN

In the experiment we used 4 triangle meshes, each consisting of about 100K vertices. The Max-Planck and the Human Head models are both natural models acquired through laser scanning of physical objects, and between them the Max-Planck model has more geometric information. The Sphere and the Cone are synthetic models created by CAD software. The Sphere model consists of almost equilateral triangles, while the Cone mostly comprises long skinny triangles. Overall, the choice of the models of the experiment aimed at establishing the relationship between the discrimination threshold on the one hand, and two shape related factors on the other, that is, the amount of geometric information carried by the mesh, and the average shape of its individual triangles.

For each of the 4 original unquantised meshes, 10 different quantised meshes were produced, one for each integer quantisation level, from 8 bits per vertex coordinate to 16 bits per vertex, while the quantisation level of 20 bits per vertex was also used.

Rendered images were produced from these meshes and were presented to the participants as the stimuli of the experiment, see Figure 2. The high specular component of the rendering method resulted into surfaces with a characteristic glossy appearance. As it was shown in our previous experiment,[2] quantisation artifacts are easier to detect on high specularity renderings, because they reveal better the normal perturbations of the underlying mesh, which normal perturbations are considered the main source of visual degradation in a mesh.[3]

The experiment was conducted in Saudi Arabia, in October 2020, over a period of 25 days, with 20 participants in total. The stimuli were presented on a computer screen MacBook Pro with a resolution of 2560 × 1600 pixels. The screen width and height were 30.41 and 21.24 centimeters, respectively. The observer viewed the screen from a distance of 50 cm. The room, where the experiment took place, has a natural light and quit so no distraction could affect the process. The stimulus size on the screen was 1280 × 725 pixels.

Discrimination thresholds were measured using a two-alternative forced-choice method. On each trial, the original, unquantised, stimulus and the unquantised one were presented side by side on the screen for 4000 ms. The observer then used the computer keyboard or mouse to indicate which of the two stimuli has the highset quality. See Figure 3 for the interface of the experiment. The next trial started after the response.

Each quantisation level was repeated 10 times during the experiment. As there were four different models and 10 quantisation levels for each model, there were a total of 4*10*10=400 trials in the experiment for each observer. The order of models and quantisation levels was randomised across trials. Due to a programming error, the left/right order of the original and the quantised stimulus was not randomised, but switched after each trial. That is, on every other trial the original was on the left, and on every other, on the right.

Before the experiment the participants were given instructions on how to conduct the experiment. They were then allowed to test the interface and practice the task. The models in this practice experiment were different from those of the main experiment.

Following,[2] discrimination thresholds are obtained by fitting to the data the psychometric function

$$f(x; \alpha, m, s) = 1 - \alpha \cdot \Phi_{m,s}(x) \quad (1)$$

where x denotes quantisation level, $\Phi_{m,s}$ is the cumulative Gaussian distribution with mean m and standard deviation s, and α is a third free variable of the model, representing the asymptotic probability of a wrong answer for high quantisation levels. A Matlab program was implemented to fit $f(x; \alpha, m, s)$ to a given set of observations by a maximum likelihood estimation (MLE) of its three free variables α, m, s.

Notice that in a two-alternative forced-choice experiment, one would normally expect the value of α to be equal to 0.5, reducing the number of free variables to two. That is, for increasingly higher levels of quantisation, and as quantised and unquantised models become indistinguishable, one would expect the probability of correct answer to tend to 0.5. Nevertheless, here, α was added to the model as a free variable to be estimated along m and s. This is because the left-right order of the two stimuli was not randomised in the experiment (see above), and thus it remains possible that some bias in the observer's responses leads to a value of α different from 0.5. In that case, if the value of α was forced to 0.5, the estimates of the other parameters, which are of main interest to us, would have been biased. In particular, in order to get a good estimate of the threshold, one should also fit α.

3. RESULTS

Fig. 4 shows the MLE fitted curves for the four models of the experiment, and Table 1 summarises the estimates of the variables α, m, s. Since a Gaussian probability distribution function has its maximum at m, the corresponding cumulative probability distribution has at m its inflection point, which is also the point where the maximum of its derivative is obtained. Thus, m corresponds to the level of quantisation where the probability of a correct answer takes a value exactly at the middle between its absolute maximum of 1 and its asymptotic minimum $(1 - \alpha)$. Therefore, m is the best, in the maximum likelihood sense, estimate of the discrimination threshold. As

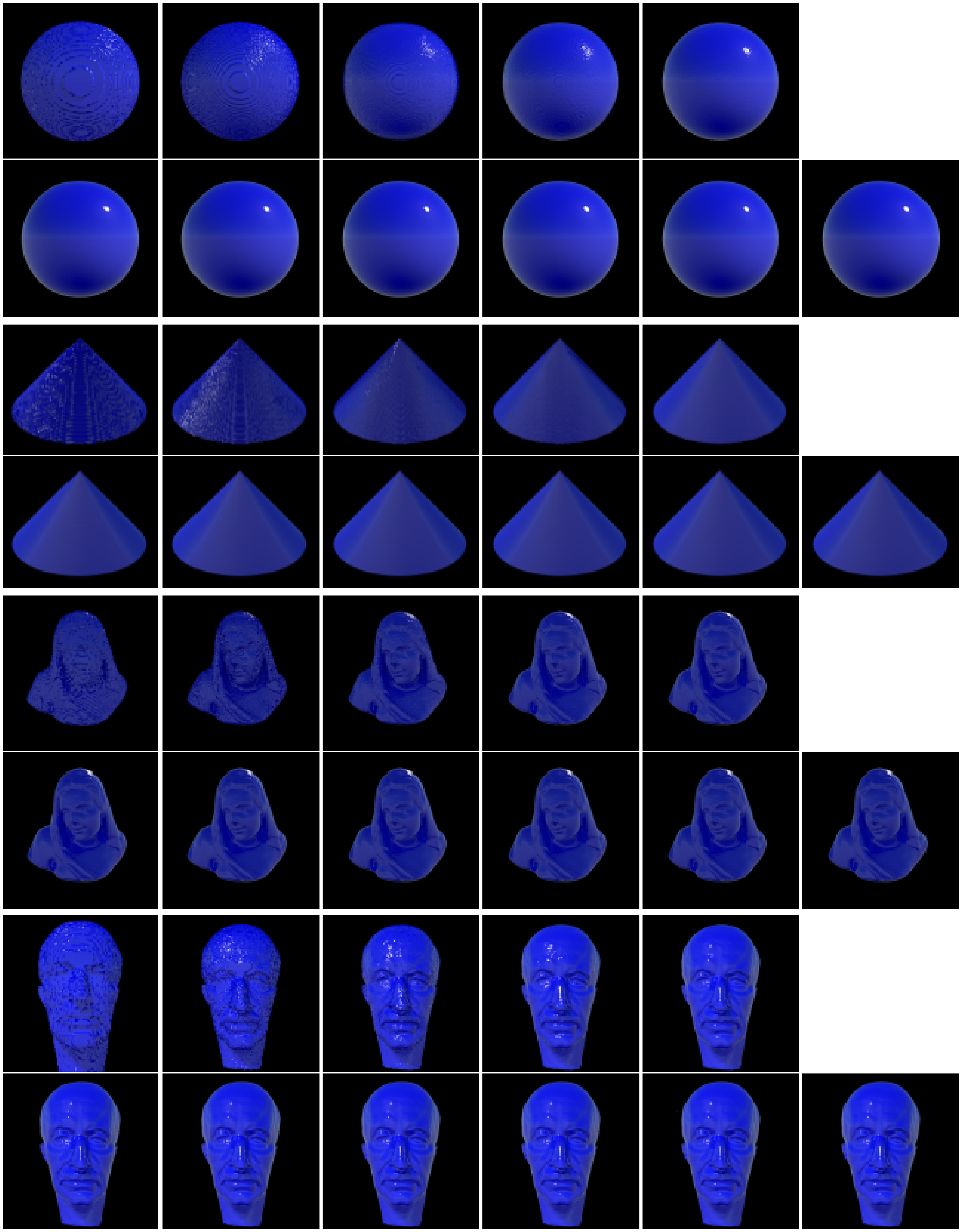

Figure 2. For each model, the top row shows meshes quantised at levels 8-12. The bottom row shows meshes quantised at levels 13-16, 20 and the original unquantised model.

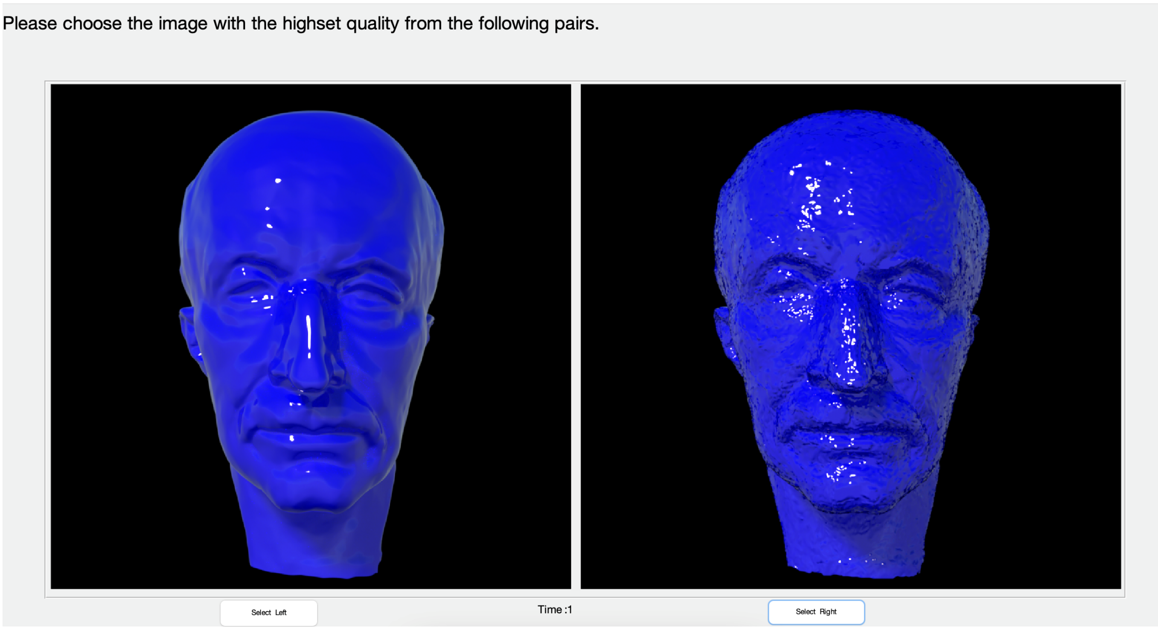

Figure 3. The interface of the experiment.

expected, the values of m for the four models follow the inverse order of the values of the mean probability of choosing the unquantised model.

As seen in Fig. 4, the psychometric functions do not asymptote at 0.5. The fit values for α were systematically smaller than 0.5 indicating that, counter-intuitively, when there was very little difference between the two stimuli, the observer chose the quantised one as having a better quality. It is very unlikely that this is a true perceptual effect, however, as this holds also for the quantisation level 20, which was practically identical with the original. It is more likely to result from non-independence of the observer's responses across trials. Several types of sequential effects are known to exist between trials in a psychophysical experiment.[26,27] Since there are not enough data to distinguish between them, we focus on the other parameters m and s.

As a measure of the amount of geometric information carried by a 3D model we use the filesize, after applying a state-of-the-art mesh compression algorithm, here the publicly available Draco software `https://google.github.io/draco/`. The use of the absolute filesizes of the compressed meshes, instead of compression ratios, is justified by the fact that, here, we are interested in an absolute measure of the amount of geometric information carried by the mesh, rather than the amount of geometric information redundancy in that mesh.

As a measure of the average quality of an individual triangle of the mesh, the average aspect ratio over all triangles of the mesh was used. The aspect ratio of a triangle was computed as the length of the shortest edge of the triangle, divided by the length of the longer. The aspect ratio of an equilateral triangle is 1, while for long skinny triangles, the aspect ratio tends to 0. The aspect ratio is one of the various element quality metrics described in.[28] We note that all the various other metrics also favour equilateral triangles against thin ones, and the results from their use in place of the aspect ratio, are expected to be similar. We also note that averaging over triangle quality metrics can be used to derive quality metrics that would apply to whole meshes.[29]

Table 1 summarises the variables from the analysis of the experimental results and the analysis of the models themselves.

The values of m in Table 1 verify our previous observations that were based on the raw probabilities of correct answer. For example, on the Sphere model, the user needs a lower increase of the stimulus level (here, the quantisation artifacts) to go from a correct to a wrong answer, as compared against the Sphere, hence the discrimination threshold is higher. We also note that the differences between the discrimination thresholds of different models are rather small, indicating that in certain application domains it could be possible to find

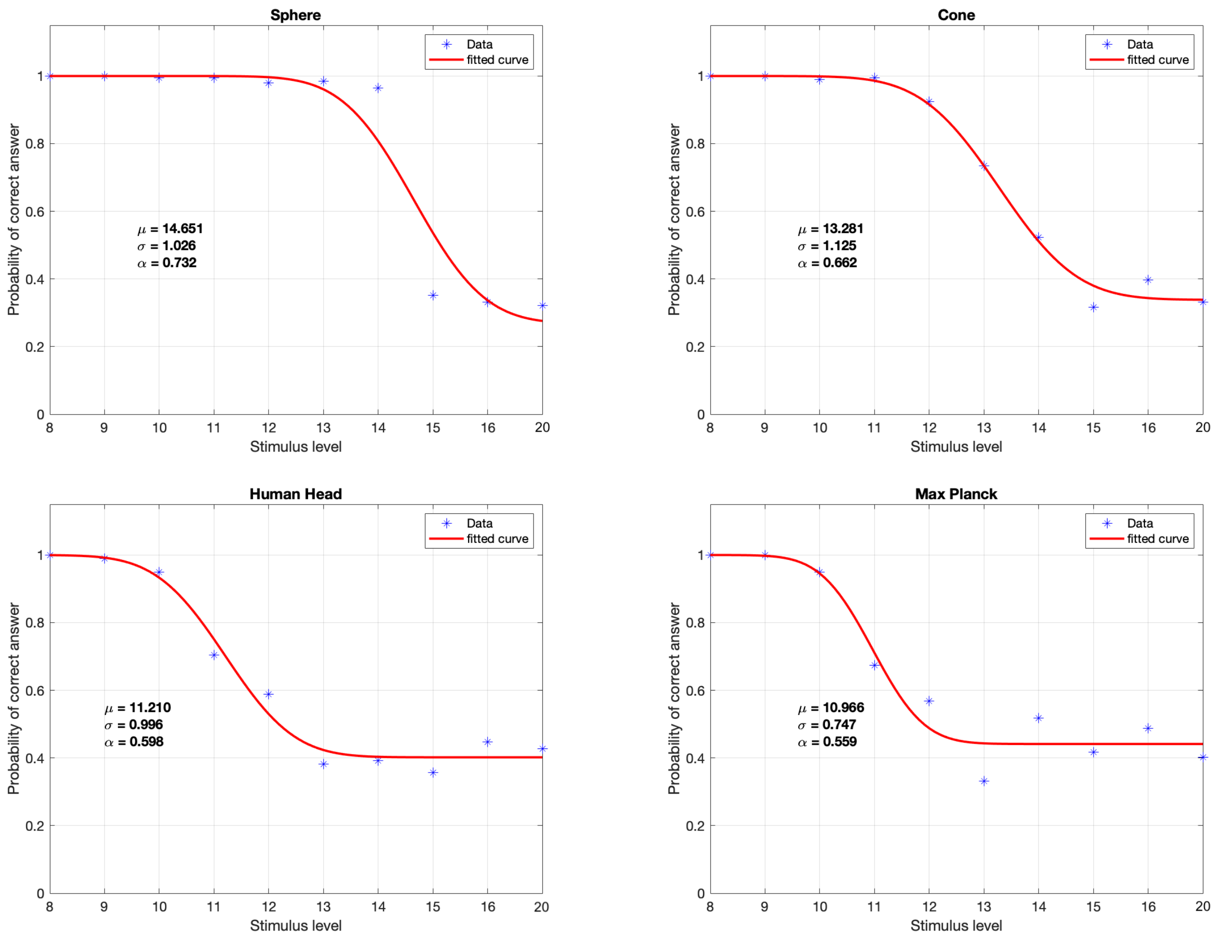

Figure 4. The Maximum Likelihood Estimated psychometric function for each of the models of the experiment.

Table 1. The parameters of the fitted psychometric function (top three rows), the slope at the inflection point computed as α/s (fourth row), and geometric characteristics of the original meshes (bottom three rows).

	Sphere	Cone	Head	Max
m	14.651	13.281	11.210	10.996
s	1.026	1.125	0.996	0.747
α	0.732	0.662	0.598	0.559
slope = α/s	0.713	0.588	0.600	0.748
# Mesh triangles	307.520	314.400	216.928	199.996
Compressed filesize	310.848	342.020	402.932	423.724
Mean aspect ratio	0.9635	0.3913	0.8043	0.6401

universal discrimination thresholds that will also be efficient in terms of memory usage. That is, one can use universal discrimination thresholds that are not, unnecessarily high, e.g. 16 or 24, rather than 32 bits per vertex.

The steepness of the transition from the highest to the lowest value of the psychometric function depends on its slope at the inflection point. For the type of the psychometric function employed, this slope is proportional to α/s, see,[30] the values of which are reported in the fourth row of Table 1. Notice that the Sphere and the Max-Planck models have higher slopes, and thus sharper discrimination thresholds than the Cone and the Head models. However, it is clear that the experiment does not provide enough evidence, not even for a qualitative study of the issue.

The values of α show a systematic bias in favour of the models quantised at a high quantisation level, and against the unquantised model. As mentioned in Section 3, this could in part be explained by a flaw in the implementation of the experiment, that is, using alternation rather than the randomisation in the relative positions of the two stimuli within the interface.

3.1 Correlations between discrimination thresholds and mesh geometry

The results show a clear correlation between the discrimination threshold and the filesizes of the compressed meshes. Quantitatively, a correlation coefficient of $r = -0.9915$, with $p = 0.0085$, was computed, indicating a strong inverse correlation, with a high statistical significance. The result is intuitive, as one would expect that, given the quantisation level, an observers' ability to detect quantisation artifacts at that particular scale would depend on the amount of geometric information the artifacts are embedded in. That is, at a given level of quantisation, artifacts should easier to detect over meshes that carry little geometric information, such as the sphere, rather than on meshes with more geometric information such as the Max-Planck.

The experiment used both natural meshes, acquired by laser scanners and carrying large amounts of information, and synthetic meshes that carried less information. We note that the correlation between discrimination thresholds and filesize of the compressed mesh is evident both across the two mesh types, as well as within each one of them. Regarding the comparison across mesh types, the synthetic meshes, carrying less information, had higher discrimination thresholds than the natural ones. Within the synthetic mesh type, the Sphere, carrying the least geometric information, had higher threshold than the Cone. Within the natural mesh type, the Max-Planck, having more prominent features and carrying more geometric information, had as expected a larger compressed filesize than the Head, and eventually a lower discrimination threshold.

We consider this correlation as the most significant result of our experiment. Notice that, generally, and especially in signal theoretic studies, the ratio of the carrier strength to the amount of noise is considered an important measure of the expected performance of a system. However, to the best of out knowledge, it is the first time that a similar observation is made in such a setting, that is, regarding the visual perception of a mesh as established by a psycho-physical experiment on the one hand, and the amount of geometric information carried by that mesh, as measured by its compressed filesize, on the other. On the other hand, the relationship between performance in a psycho-physical task and the amount of geometric information carried by the stimuli, is a phenomenon that can be observed in various contexts, and has been studied extensively. One of the

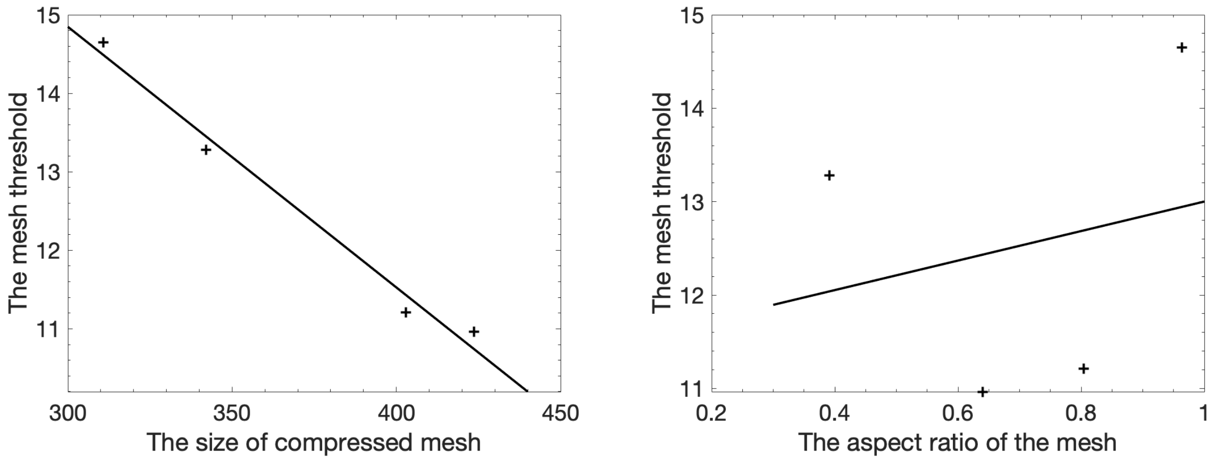

Figure 5. Scatter plots of the compressed mesh filesize (left) and the mean aspect ratio (right), against the discrimination thresholds. The best fitting lines are shown too.

most notable examples is the enumeration of dots on planar configurations of various degrees of symmetry and regularity.[31,32]

The analysis of the results does not show any statistically significant correlation between the discrimination thresholds and the average aspect ratio of the triangles of the mesh. Specifically, the correlation coefficient was computed at just $r = 0.2198$, with $p = 0.7802$.

The lack of a statistically significant correlation could be interpreted as an indication of the unsuitability of that mesh quality metric to predict mesh discrimination thresholds. Indeed, the metric averages aspects ratios over all the triangles of the mesh, including triangles in the non-visible part of the mesh. On the other hand, it could be the case that some users were evaluating some meshes by focusing their attention on specific parts of them. Especially, the parts of the mesh which, depending on the mesh position and orientation, and the lighting conditions, reflect most of the incident light directly on to the camera.

Alternatively, we cannot altogether exclude the possibility that there is such a correlation, which however must be weaker than that between the compressed filesize and the discrimination threshold. This possibility could be investigated by a follow-up experiment with meshes that have similar compressed filesizes and different mean aspect ratios.

Figure 5 shows the scatter-plots of the compressed filesizes and the mean aspect aspect ratios, respectively, against the discrimination thresholds. The tightness of the best fitting line in the first scatter-plot illustrates the high correlation between the compressed filesizes and the discrimination thresholds.

4. CONCLUSIONS

This paper presented the results of a two-alternative forced-choice psychophysical experiment, aiming at studying the quantisation thresholds, below which the degradation of the visual quality of the mesh by the quantisation artifacts can be detected. Our main finding is that there is a strong inverse correlation between the discrimination threshold of a mesh and its filesize after compression. A possible link between discrimination thresholds and the quality of the mesh triangles, as measured by the mean aspect ratio, was also studied, but no significant such correlation was detected.

The main limitations of the experiment presented in this paper stem from its relatively small size. In the future, we plan to use the results of this experiment to inform the design of a larger experiment, aiming at confirming the correlation between the amount of geometric information in a mesh, as measured by the filesize of the compressed mesh filesize, and the discrimination thresholds, and clarifying its scope. In particular, in this follow-up experiment sparser meshes with fewer triangles will also be included, as well as non-uniform meshes with some areas more densely triangulated than others. Finally, in a second follow-up experiment we would like to study possible relations between the amount of geometric information in the mesh and the perceptibility of other types of noise and artifacts embedded into it, for example, Gaussian noise, or subdivision artifacts.

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