Rainfall induced deformation on unsaturated collapsible soils

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Abstract.

The stability of transportation infrastructure can be affected by seasonal rainfall infiltrated adjacent excavations and natural slopes. The paper investigates the rainfall induced deformation of an anchored excavation constructed in unsaturated soils. An advanced constitutive model based on the theory of bounding surface plasticity is employed to predict the behaviour of partially saturated soils. The collapse characteristics of unsaturated soils due to wetting are captured in the model by introducing a suction dependent hardening law. The proposed model has been implemented in a finite difference code with a two-phase flow and deformation formulation. In the numerical algorithms, the validity of the effective stress principle in unsaturated soils is emphasized and the coupling between various phases of porous media has been considered. The problem of an anchored excavation subjected to rainfall is then simulated with the implemented model. The lateral deformation of the supported excavation during the infiltration of water into unsaturated collapsible soils is obtained and compared with the case where the plastic collapse is prevented. Finally, the effect of anchors on minimizing and changing the mode of the deformation is explored.

Keywords: Rainfall, unsaturated soils, constitutive model.

1 Introduction

The infiltration of rainfall into slopes and excavations poses a serious threat to the safety and stability of infrastructure built on them. Due to the extensive presence of unsaturated soils across the globe, research on rainfall-induced deformation has been an important development in unsaturated soil mechanics. Rainfall seepage can increase the pore water pressure inside unsaturated soils, leading to a drop in the effective stress thus generating deformation. On the other hand, the infiltration process can substantially alter the hydraulic properties of soils due to the change of the porosity. To explain properly the above hydro-mechanical process, the employment of a coupled flow and deformation model is required [1].

The reduction of the suction during seepage not only creates recoverable deformation via changing the effective stress but also can trigger irreversible wetting-induced collapse. The latter effect of the rainfall cannot be explained successfully by the constitutive models available for saturated soils. With the aid of a coupled multiphase plasticity model, Cho and Lee [2] investigated the behaviour of a slope subjected to rainfall infiltration. The extended Mohr–Coulomb failure criterion was adopted for the plasticity modelling and the effect of the change in porosity due to the suction was addressed by a state surface suggested by Lloret and Alonso[3]. This model has been used by Alonso [4] to assess the stability of a slope in overconsolidated clay under rainfall. The mixture theory for multi-phase porous media with an effective stress based plasticity model has been used by Oka et al. [5] to analyse the problem of the seepage through river embankments. Adopting the extended cam–clay model for solid phase, Borja and White [6] conducted a numerical analysis for rainfall induced deformation of a steep slope and determined the failure mechanism. Recently, Zheng et al [7] implemented a preliminary unsaturated model (the basic Barcelona model) in a finite difference code. They analysed a steep slope subjected to the rainfall infiltration and predict a wetting-induced plastic deformation.

Most of the fully coupled formulations cited above incorporated preliminary constitutive model for the solid phase and therefore fail to explain comprehensively the collapse behaviour of unsaturated soils. Also, incorporating a constitutive model with independent stress variables will reduce the applicability of the models since a time-consuming process is required for the determination of the model parameters. To overcome these deficiencies, the paper aims at implementing an advanced plasticity model in a coupled hydro-mechanical finite difference code (FLAC). A bounding surface plasticity model is adopted for the constitutive modelling of the solid phase (Khalili et al [8], Russel and Khalili [9]). The model enjoys the validity of the effective stress principle and includes the hardening effect of suction. The problem of rainfall infiltration into an anchored excavation is then simulated to show the susceptibility of unsaturated soils to wetting collapse.

2 Effective stress principle, water retention and permeability model

From the conceptual viewpoint, the mechanical behaviour of unsaturated soils can be theoretically formulated within the principles explaining the behaviour of multiphase porous media. For single-phase media, Terzaghi’s effective stress law can describe many features of the behaviour of saturated soils. It has been shown that this theory can be generalized for two-phased porous media as

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \chi p_w \boldsymbol{\delta} + (1 - \chi) p_g \boldsymbol{\delta} = \boldsymbol{\sigma}_{\boldsymbol{net}} - \chi s \boldsymbol{\delta} \tag{1}$$

Where σ' is what we shall call "effective stress", $\boldsymbol{\sigma}$ is total stress, $\boldsymbol{\delta}$ stands for the identity vector, χ is the "effective stress" parameter, p_w and p_g are wetting (water) and non-wetting (air) pressures, $s = p_g - p_w$ is the matric suction and $\boldsymbol{\sigma_{net}} = \boldsymbol{\sigma} + p_g\boldsymbol{\delta}$ is the net stress. The "effective stress" parameter is a key variable in the above equation. In this study, the wetting degree of saturation, S_w, has been regarded as the "effective stress" parameter despite the fact that more elaborate relationships, which can better predict the results of experimental data, can be utilized for this parameter (see [10]). The degree of saturation can be expressed in terms the matric suction, which is recognized as the water retention model. Following the work of the van Genuchten [11], the effective degree of saturation can be linked to matric suction as

$$S_{eff} = \left[1 + (\frac{s}{p_0})^{1/(1-a)}\right]^{-a} \tag{2}$$

Where p_0 and a are two material parameters, the former has a close link to the suction related to air-entry or air expulsion values and S_{eff} is effective degree of saturation defined as $S_{eff} = (S_w - S_r^w)/(1 - S_r^w)$, where S_r^w is residual wetting fluid saturation. In two-phase porous media, the relative permeability laws for the wetting and non-wetting phases can be correlated to the effective degree of saturation. Aligned with the work of van Genuchten [11], the following expressions hold for the relative permeability models

$$k_r^w = S_{eff}^b \left[1 - \left(1 - S_{eff}^{1/a}\right)^a\right]^2 \tag{3}$$

$$k_r^g = (1 - S_{eff})^c \left[1 - S_{eff}^{1/a}\right]^{2a} \tag{4}$$

Where a , b and c are model parameters and k_r^w and k_r^g are relative permeability of the wetting and non-wetting phases respectively. Having defined the model's empirical relationships, a theoretical framework based on the principle of the mass and momentum balance can follow to obtain the fluid constitutive laws (see [8]). To obtain a fully coupled flow and deformation model, the employment of the momentum balance equation for the solid phase is also necessary. The constitutive model for the solid phase can finally be obtained by introducing a stress-strain relationship capturing the essential features of the behaviour of unsaturated soil. In this study, an advanced model based on the theory of the bounding surface plasticity is used as the constitutive relationship of the solid phase.

3 Bounding surface plasticity model

The bounding surface is the locus of the possible state of stress in the stress space. This surface can be determined from the response of the unsaturated soils in the loosest state. A flexible shape is adopted for the shape of the bounding surface to accommodate versatile behaviour of unsaturated soil

$$F\left(\bar{p}',\bar{q},\bar{\theta},\bar{p}'_c\right)=\left[\frac{\bar{q}}{M_{cs}\left(\bar{\theta}\right).\bar{p}'}\right]^N-\frac{\ln\left(\bar{p}'_c/\bar{p}'\right)}{\ln\left(R\right)}=0 \tag{5}$$

In which $\bar{p}'_c$ adjusts the size of the bounding surface and N and R are material parameters. Further, $\bar{p}',\bar{q},\bar{\theta}$ stands for the mean “effective stress”, deviatoric stress and Lode angle on the bounding surface. It is noted that over-bar indicates parameters given on the bounding surface. It is clear from equation (5) that a three-dimensional shape is proposed for the bounding surface. The slope of the critical state line (CSL) has been denoted by $M_{cs}\left(\bar{\theta}\right)$ in the equation (5) , which can be expressed in terms of lode angle

$$M_{cs}\left(\bar{\theta}\right)=M_{max}\left[\frac{2\alpha^4}{1+\alpha^4-\left(1-\alpha^4\right)sin\left(3\bar{\theta}\right)}\right]^{1/4} \tag{6}$$

Where $\alpha=M_{max}/M_{min}$ with M_{max} and $M_{\min}$ are the slope of CSL at triaxial compression and extension. The relationship between M_{max} and $M_{\min}$ is $M_{\min}=3M_{\max}/(M_{\max}+3)$. The movement of the stress point has been enforced to lie on the loading surface. In this study, an identical shape has been adopted for the loading surface as for the bounding surface assuming that both surfaces are homologous about a centre of homology. The origin of the stress is taken as the centre of homology in this study. Specifying the direction of the plastic strain is another ingredient of the plasticity model involved. Following the work of Russell and Khalili [9], a non-associated plastic potential is selected

$$g\left(p',q,\theta,p_0\right)=q+\frac{AM_{cs}\left(\theta\right)p'}{A-1}\left[\left(\frac{p'}{p_0}\right)^{A-1}-1\right] for\, A\neq 1$$
$$g\left(p',q,\theta,p_0\right)=q+M_{cs}\left(\theta\right)p'ln\left(\frac{p'}{p_0}\right) for\, A=1 \tag{7}$$

In which A is the material parameter and p_0 indicates the size of the plastic potential surface. The last ingredient of the bounding surface plasticity is to identify hardening modules. Following the usual procedure in the theory of the bounding surface plasticity, the hardening module can be decomposed into two terms

$$h=h_b+h_f \tag{8}$$

Where h_b is the hardening modulus on the bounding surface and h_f is the hardening modulus depending on the distance of the stress point from an conjugate point on the bounding surface. Proposing a hardening modulus as a function of the distance to the bounding surface ensures that smooth elasto-plastic behaviour can be achieved. The following functions can be taken for these moduli (see Khalili et al [8])

$$h_b=-\frac{\partial F}{\partial \overline{p}_c^{'}}\left(\frac{\partial \overline{p}_c^{'}}{\partial \varepsilon_v^p}+\frac{\partial \overline{p}_c^{'}}{\partial s}\frac{\dot{s}}{\dot{\varepsilon}_v^p}\right)\frac{m_p}{\left\|\partial F/\partial \overline{\boldsymbol{\sigma}}^{'}\right\|}$$
$$h_f=\left(\frac{\partial \overline{p}_c^{'}}{\partial \varepsilon_v^p}+\frac{\partial \overline{p}_c^{'}}{\partial s}\frac{\dot{s}}{\dot{\varepsilon}_v^p}\right)\frac{p'}{\overline{p}_c^{'}}\left(\frac{\overline{p}_c^{'}}{p_c^{'}}-1\right)k_m\left(\eta_p-\eta\right) \tag{9}$$

In which $\dot{\varepsilon}_v^p$ is the increment for the plastic volumetric strain, k_m is a material parameter and $m_p=\frac{\partial g}{\partial p'}/\left\|\frac{\partial g}{\partial \boldsymbol{\sigma}'}\right\|$. Also, $\eta=q/p$ is the stress ratio, $\eta_p=\left(1-2(\upsilon-\upsilon_{cs})\right)M_{cs}\left(\theta\right)$ is the slope of the peak strength line, υ is the specific volume defined as $\upsilon=1+e$ and υ_{cs} is the specific volume at CSL. It is obvious that both volumetric strain and matric suction can generate plastic deformation. This is a unique feature of the model for the simulation of the problems involving the effect of suction change and volumetric deformation. The simultaneous effect of the suction and volumetric strain on the evolution of the size of the bounding surface is predicted by

$$\bar{p}'_c\left(\varepsilon_v^{\mathrm{p}}, s\right) = \bar{p}'_{ci}\gamma\left(s\right)\exp\left(\frac{\upsilon_i \Delta\varepsilon_v^{\mathrm{p}}}{\lambda\left(s\right)-\kappa}\right) \tag{10}$$

With

$$\gamma\left(s\right) = \exp\left(\frac{\mathrm{N}\left(s\right)-\mathrm{N}\left(s_i\right)}{\left(\lambda\left(s\right)-k\right)} - \frac{\left(\lambda\left(s\right)-\lambda\left(s_i\right)\right)}{\left(\lambda\left(s\right)-k\right)}\ln\left(\bar{p}'_{ci}\right)\right) \tag{11}$$

Where υ_i and $\bar{p}'_{ci}$ are the initial specific volume and hardening parameter, k is the slope of unloading-reloading curve on $\upsilon \sim \ln p'$ plane. Also $\mathrm{N}\left(s\right)$ and $\lambda\left(s\right)$ are the intercept and slope of limiting isotropic compression line (LICL) on $\upsilon \sim \ln p'$ plane at the suction s, $\mathrm{N}\left(s_i\right)$ and $\lambda\left(s_i\right)$ are corresponding parameters at the initial suction s_i. LICL can be obtained from the result of the isotropic compression test performed on the loosest soil sample.

4 Rainfall induced deformation in an anchored excavation

The proposed constitutive model has been implemented in an explicit finite difference code (FLAC) as a user defined constitutive model. To show the applicability of the model in predicting the response of collapsible soils, the problem of an excavation under rainfall infiltration is simulated. The geometry of the problem is depicted in Figure 1. The hydraulic properties of the soil are available in Table 1. In this table, μ_w and μ_g are dynamic viscosity of wetting and non-wetting phases respectively. Also, K_w and K_g are bulk modules for wetting and non-wetting phases and κ_p is saturated mobility coefficient defined as the ratio of intrinsic permeability over dynamic viscosity of the water phase. The material properties for the solid skeleton are defined in the Tables 2 and 3. In these tables, the elastic parameters of the model(e.g. κ and ν) are determined through calculating the initial tangent stiffness obtained from the stress-strain curve under shearing. The parameters related to the critical state condition(M_{cs} and A) are obtained in the ultimate state of soils achieved in the residual state. The analyse of the behaviour of soils under isotropic loading with various suction can indicate the, $\lambda(s)$ and $N(s)$, from which the model parameters R have been determined. k_m is a fitting parameter adjusting the hardening modules during unloading reloading cycles. The parameter N can also be determined through the shape of bounding surface obtained in the loosest state of saturated soil.. Finally, the mechanical properties for anchors are introduced in Table 4.

Table 1. Hydraulic properties for soil

P_0(kPa)	a	*b*	c	S_{res}
7	0.58	0.5	0.33	0
$\mu_w/\ \mu_g$	K_w(MPa)	K_g(Pa)	κ_p(m^2/Pa.s)	
55	2	20	1e-8	

Table 2. Mechanical properties of soil

κ	ν	M_{cs}	N	R	A	k_m
0.05	0.2	1.1	1.44	2	2	20

Table 3. Suction dependent LICL for soil

Suction (kPa)	$s<s_{ae}=100$	700
$\lambda(s)$	0.2	0.2
$N(s)$	3.112	3.212

Table 4. Mechanical properties for anchor

Young's modulus (GPa)	Yield strength (kN)	Cross section (m^2)
210	250	5e-4
bond stiffness (N/m/m)	bond strength (N/m)	Spacing (m)
1.5e10	8e5	5

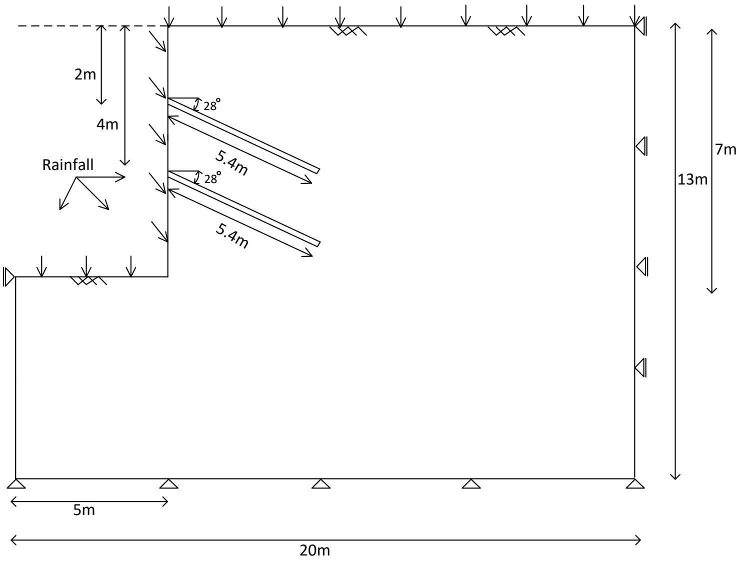

Fig. 1. The sketch of the excavation problem subjected to rainfall

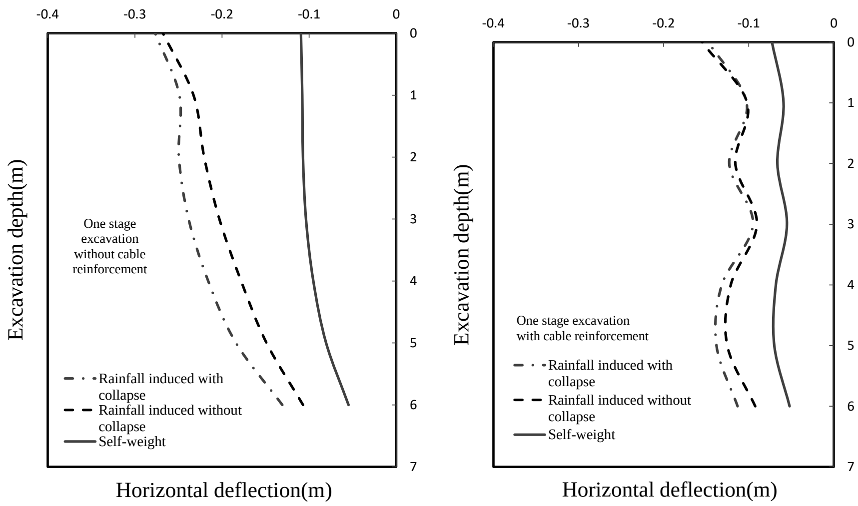

Fig. 2. The horizontal deflection of an excavation a) without the instalment of anchors b) with instalment of anchors

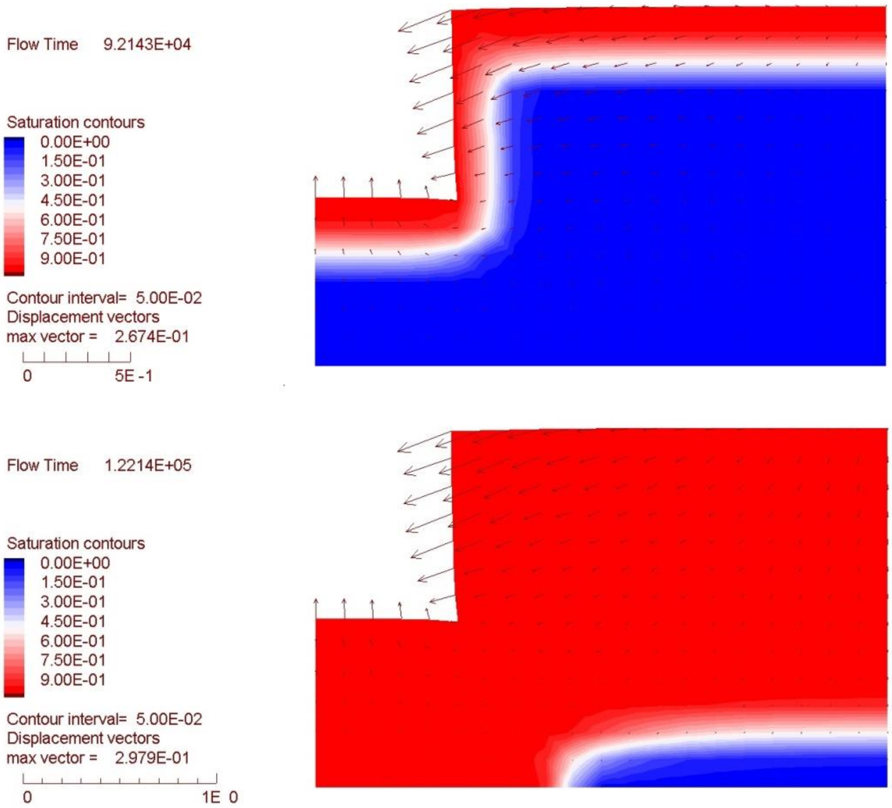

Fig. 3. The change in the degree of saturation and deformation vectors without collapse behaviour a) t=25 hours (9e4 seconds) b) t=34 hours (1.22e5 seconds)

The unsaturated soil with initial suction of 700 kPa is assumed for the soil behind the wall . The initial specific volume is assigned based on the assumption that the preconsolidation pressure for the whole layer is 1400 kPa. To simulate the effect of rainfall infiltration, zero suction was applied at the ground surface and the inner face of the excavation, which is equivalent to full saturation for the wetting phase. Also a seepage boundary condition was employed at the bottom of excavation to absorb the flow of wetting phase(boundary is impermeable until degree of saturation is 1; afterwards the pore water pressure is set to zero). It is assumed that all of the excavated area was removed in one stage. The suction dependent LICL was updated during the wetting simulation to reduce the strength of the unsaturated soils, capturing the plastic collapse behaviour.

The horizontal deflection of the excavation was obtained for the case where there was no rainfall on the top surface of the soil (the effect of the self-weight only) and compared to the excavation problem subjected to the rainfall. Also the effect of plastic collapse due to wetting was included in the simulation. This was achieved by simulating the excavation problem under single LICL (the strength corresponding to the suction of 700 kPa) and comparing the result with the case of an excavation with suction-dependent LICL. In Figure 2, the horizontal deflection of the excavation is depicted for the three cases described above. For an unanchored excavation, the increase of the horizontal deflection is predicated upon wetting. If the suction- dependent strength parameters were not involved in the analysis, the model could only predict the excess deflection due to the decrease of the effective stress. However, the full wetting-collapse behaviour can be achieved if the variation of the LICL with respect to the matric suction is included.

A similar pattern can be seen for the horizontal deflection of an anchored excavation (see Figure 2), while different modes of deformation are obtained upon the plastic collapse of the sample. It is also evident that employing anchors can reduce significantly the lateral deflection of the excavation. The change in the degree of saturation during the simulation is depicted in Figure 3 for the excavation with suction-dependent LICL. It can be seen that there is vertical settlement on the surface of the soil when rainfall penetrates into the soil. Also, large amount of the horizontal deflection occurs if the rainfall infiltrates into the soils.

5 Conclusion

The paper studies the deformation of an anchored excavation built on unsaturated soils subjected to rainfall infiltration. Unsaturated soils consist of three phases possessing independent compressibility properties. Relationships for the “effective stress” law, degree of saturation and the relative permeability are introduced, which all relies on the variation of suction as a main variable. A non-associated bounding surface plasticity model was developed to predict the behaviour of solid phase. The model offers smooth elasto-plastic behaviour within the bounding surface since a hardening modulus proportional to the distance of the stress point from the image point on the bounding surface is advised. The wetting induced collapse of unsaturated soils is captured via defining a suction dependent hardening modulus. A user defined constitutive model has been written and embedded in an existing finite difference code. With the aid of a coupled flow and deformation model, an anchored excavation under rainfall infiltration is simulated. It has been shown that noticeable lateral deflections can occur if the strength reduction due to the drop of the suction value is permitted in the model. The amount of these deflections can be decreased substantially if a proper set of anchors are installed on the side-wall of the excavation. Finally, the change of degree of saturation during the simulation has been depicted for the whole domain of the problem.

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