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Aristotle on Mathematical Pythagoreanism in the Fourth Century BCE

In this first chapter, I will attempt to describe the kinds of Pythagoreans who may have existed from the sixth through fourth centuries BCE and the philosophical activities in which they seem to have engaged by appeal to the evidence preserved by Aristotle. The goal is to identify the characteristics that distinguished the mathematical Pythagorean *pragmateia*—where we may tentatively describe a *pragmateia* here (for Aristotle) as both the object of a philosophical inquiry and the treatment of the same object—from the *pragmateia* of the rival acousmatic Pythagorean brotherhood in Magna Graecia. This goal is part and parcel of the larger project that will occupy the entirety of this book: to trace the history of mathematical Pythagoreanism from a variety of informed ancient perspectives. My claim in this chapter is that Aristotle, especially in *Metaphysics* A and the lost writings on the Pythagoreans (preserved in a fragmentary state without significant modifications in Iamblichus's work *On the General Mathematical Science*),[1] establishes this distinction by appeal to the divergent philosophical *methodologies* of each group: the mathematical Pythagoreans, who are the same as the "so-called Pythagoreans" in *Metaphysics* A, employ superordinate[2] mathematical sciences in establishing something that approximates demonstrations that explain the "reason why" (τὸ διότι) they hold their philosophical positions, whereas the acousmatic Pythagoreans, who are distinguished from the "so-called" Pythagoreans in *Metaphysics* A, appeal to basic, empirically derived "fact" (τὸ ὅτι) in defense of their doctrines. Furthermore, I suggest, Aristotle criticizes the *pragmateia* of the mathematical Pythagoreans for improper methodological procedure: while the

1. I refer to Aristotle's "works on the Pythagoreans" on the grounds that we cannot know for sure what work Iamblichus was using to extract his descriptions of the Pythagoreans. Titles are attested for *On the Pythagoreans* (one book), *Against the Pythagoreans* (one book), *On the Philosophy of Archytas* (three books), and *Summary of the Timaeus and of the Works of Archytas*.

2. I employ the term "superordinate" to refer to those sciences Aristotle considered superior to the "subordinate" or "one beneath the other" (θάτερον ὑπὸ θάτερον) sciences (*APo.* 1.7, 75b15–16), following Johnson 2009.

demonstrations offered by the mathematical Pythagoreans represent a significant philosophical innovation over the uncritical reflection on the so-called "facts" by the acousmatic Pythagoreans, the mathematical Pythagoreans' activity of hasty assimilation across categories leads to confusions in logic and metaphysics. Analysis of the extant fragments of Philolaus of Croton (among others) gives evidence for the kind of approach to understanding the universe that Aristotle associates with the mathematical Pythagoreans, and it becomes likely that the targets of Aristotle's disapproval are those Pythagoreans who undertook to perform basic demonstrations of the Pythagorean definitions of things as preserved in the *acusmata* attributed to Pythagoras. It becomes possible, then, to inquire further as to whether Aristotle's classification might have any value for a reconstruction of the philosophical methodologies of earlier Pythagoreans in the first half of the fifth century BCE.

The Pythagoreans of the fifth century BCE probably did not see themselves as a community unified by philosophical and political doctrines. Rather, insofar as we can reconstruct their history, there arose an internal conflict among the Pythagoreans who were living in the southern part of Italy, which appears to have effected a split between the ascetic Pythagoreans who lived in the western part of Italy (and fled to Asia Minor) and the intellectualist Pythagoreans who occupied the eastern part of the Italian peninsula, near Tarentum. Differences in approach to the philosophical "life" and its activities can already be detected in the comic fragments that survive from the early part of the fourth century BCE, as Christoph Riedweg has shown.[3] With Aristotle, I suggest, we find a rather elaborate account of the division of the early Pythagoreans into two groups—traditionalist acousmatics (*οἱ ἀκουσματικοί*) and progressive mathematicians (*οἱ μαθηματικοί*). What the terms "acousmatic" and "mathematical" mean will require a careful examination of Aristotle's descriptions, a project that will occupy chapters 1 and 2.[4] While most modern scholars have

3. Riedweg 2005: 108–109.

4. It is extremely difficult to correlate the bifurcation into "acousmatic" and "mathematical" Pythagorean with the tripartite subsections that developed in the Hellenistic world (*σεβαστικοί, πολιτικοί, μαθηματικοί*). Délatte (1922: 22–28) took seriously the possibility of the tripartite organization, to which earlier and later traditions as well as the so-called Hellenistic pseudo-Pythagorean writings adhere closely. Burkert (1972: 193 n. 6) suggests that the triad is a chronological grouping that aligned with the terms *Πυθαγορικοί, Πυθαγόρειοι, Πυθαγορισταί* and corresponded with the "pupils, pupils of pupils, external advocates (*ἔξωθεν ζηλωταί*)" (Anon. Phot. 438b = Thesleff 237.7–12), and whose philosophical interests in Aristotelian terms are associated respectively with theology, human affairs (i.e. politics), and mathematical sciences, including geometry and astronomy. I suspect that these chronological associations are all developed, at least in some way, out of the historical writings of Timaeus of Tauromenium, whose treatment of Pythagoreanism I will discuss in chapter 3. Zhmud (2012: 183–185) considers all these distinctions to be dated much later, probably from the first century CE.

been willing to accept the classifications of acousmatic and mathematical Pythagoreans of the fifth century BCE, they nevertheless assume that certain contradictory elements within their own constructed "Pythagoreanism" might be misinterpretation or confusion on the part of ancient critics like Aristoxenus, Dicaearchus, or Timaeus,[5] all Hellenistic commentators whose accounts at least partially derive from the descriptions of Pythagoreanism in the writings of Aristotle.[6] I would like to present an alternative account. Since Aristotle, our most comprehensive early source for a history of Pythagoreanism, differentiated two groups of Pythagoreans along methodological lines (or so I will argue), we should admit the possibility that these apparently contradictory elements in our own reconstruction reflect *actual* divisions within the community. Indeed, the primary criterion for distinguishing acousmatic from mathematical Pythagoreans, as I will show, is each group's *pragmateia* (*πραγματεία*), a term that must be further contextualized in order to make sense of precisely how Aristotle draws the line.

ARISTOTLE ON THE *PRAGMATEIAI* OF THE PYTHAGOREANS

It is my contention that Aristotle differentiates two groups of Pythagoreans according to the *pragmateia* of their respective philosophies. What does the term *pragmateia* mean for Aristotle? It will be useful to start with an operating definition, which can then be developed in the course of our argument: in Aristotle's usage, the *pragmateia* of a philosopher or philosophical group is both the *object* of their philosophical inquiry and the unique *treatment* of that object in their philosophy.[7] Some possible meanings for Aristotle listed in *LSJ*: "system" (*Metaph.* 1.6, 987a30 and 1.5, 986a8), "philosophical argument or treatise" (*Top.* 1.1, 100a18 and 1.2, 101a26; *Phys.* 2.3, 194b18; *EN* 2.2, 1103b26), and "subject of such a treatise" (*Phys.* 2.7, 198a30). Similarly, Bonitz (1970) lists several possible meanings, among which we see: *rei alicuius tractatio via ac ratione instituta* (*Pol.* 3.1, 1274b37), *interdum non tam tractationem rei quam rationem rei tractandae* (*Rh.* 1.15, 1376b4), or even *quaestio* (*APo.* 2.13, 96b15). We can assume

5. Here I refer to the historian Timaeus of Tauromenium, who is not to be confused with Timaeus of Epizephyrian Locri, the fictional eponymous authority in Plato's dialogue.

6. Most recent scholars accept the distinction between acousmatic and mathematical Pythagoreans as original with Aristotle, e.g. Burkert (1972: 192–207), Huffman (1993: 11–12 and 2010), Kahn (2001: 15), McKirahan (1994: 89–93), and Riedweg (2005: 106–108); an exception is Zhmud (2012: 169–206), who wonders whether the division is original with Nicomachus, but nevertheless accepts the basic terminology along these lines.

7. Some other scholars' definitions of Aristotelian *pragmateia* are "philosophic activity" (Burkert/Minar 1972: 194) and "enterprise" (Steel 2012: 181). Unfortunately, Aristotle nowhere explicitly defines *pragmateia*.

some semantic overlap, in the sense that for Aristotle, there was a fluid relationship between these meanings. *Pragmateia* is apparently first used in a technical manner by Archytas of Tarentum, who posits it as the "treatment" or "investigation into" an object of mathematics:

> Logistic [ἁ λογιστικά] seems to be far superior indeed to the other arts in regard to wisdom, and in particular [it seems] to deal with [πραγματεύεσθαι] what it wishes more clearly [ἐναργεστέρω] than geometry. Again in those respects in which geometry is deficient, logistic puts demonstrations into effect [ἀποδείξιας ἐπιτελεῖ] and equally, if there is any *pragmateia* of shapes [εἰ μὲν εἰδέων τεὰ πραγματεία], [logistic puts demonstrations into effect] with respect to what concerns shapes as well.
>
> (Archytas F 4 Huffman = Stobaeus *Proem*; translation after Huffman)

Archytas seems to use the abstract term *pragmateia* as well as the verb *pragmateuesthai* to refer to both the object of philosophical investigation and the treatment suitable to that object. This usage is in contrast to that of Plato, where *pragmateia* more generally means "the business of" (e.g. *Grg.* 453a2–3, *Theaet.* 161e4)[8] without any technical philosophical usage. It thus becomes possible that Aristotle inherited this special use of *pragmateia* and terms related to it from Archytas himself.[9] The idea that Aristotle might have adopted the technical terminology for the categorization of objects of philosophy and particular treatment of those objects from a Pythagorean is significant, since, as I will argue, Aristotle himself uses the term *pragmateia* as a marker that establishes characteristic distinctions between acousmatic and mathematical Pythagoreans according to the treatment of the objects of their philosophical inquiry.[10] The larger implications of the difference between the *pragmateiai* of the mathematical and acousmatic Pythagoreans have a direct significance for this study,

8. Noted by Huffman (2005: 251).

9. Still, there is one place (*R.* 7, 528d1–3) where Plato uses the term *pragmateia* in relation to mathematics. Glaucon asks Socrates if the "geometry" is to be considered the "study of the plane" (τοῦ ἐπιπέδου πραγματεία). In the context of Plato's criticisms of Pythagoreanism, especially of Archytas, it is probable that Glaucon is using a term inherited from Pythagorean mathematics here.

10. That is, if we should consider Archytas to have been a Pythagorean. I count him as one, at least in a conditional sense, for reasons I lay out in chapter 3. Huffman has inferred from the fact that Aristotle wrote three books on Archytas and two books on the Pythagoreans, and from the fact that Aristotle never calls Archytas a "Pythagorean," that Archytas's "importance was not limited to the Pythagorean tradition" (2005: 128).

since, as I will show, the figure credited with establishing the distinctive *pragmateia* of the mathematical Pythagoreans, Hippasus of Metapontum (ca. 520?–440 BCE?), may have also played a central role in the political factionalization that occurred in the Pythagorean community in the second quarter of the fifth century BCE.[11]

Who was this "Hippasus of Metapontum"? A substantial portion of this book will deal with this elusive and enigmatic figure, and I will begin by contextualizing him with the broader classification of the mathematical and acousmatic Pythagoreans advanced by Aristotle. The consensus view, which follows Walter Burkert in his extremely influential study *Lore and Science in Ancient Pythagoreanism* (1972), is that Hippasus of Metapontum was a mathematical Pythagorean (μαθηματικός). What is more troubling, though, is that neither Burkert nor those who follow him are sure how to define a Pythagorean μαθηματικός or his philosophical activities.[12] This provides an opportunity for us to pursue a more complete understanding of the Pythagorean μαθηματικός, especially in the light of Aristotle's classification of two types of Pythagoreans.[13] The relevant evidence for this comes in a tricky passage from Iamblichus's work *On the General Mathematical Science*, in which Iamblichus is summarizing[14] portions of Aristotle's lost works on the Pythagoreans:

> There are two types of the Italian, also called the Pythagorean, philosophy. For there were also two kinds of people who treated it, namely the acousmatics and the mathematicians. Of these two, the acousmatics were recognized to be Pythagoreans by the others [the mathematicians], but they did not recognize the mathematicians [as Pythagoreans], nor did they think that the *pragmateia* [of the mathematicians] derived from Pythagoras, but rather that it derived from Hippasus.

11. Iambl. *DCM* 25, 76.16–77.24 and *VP* 257–258, 138.14–139.9. I will discuss these specific passages more extensively in chapters 2 and 3, respectively.

12. Burkert 1972: 192–201. Similarly followed by Huffman (2005), Riedweg (2005), and Kahn (2001). Zhmud (2012: 255–258) emphasizes the role that ἀποδείξεις play in Pythagoras's teaching of the μαθηματικοί in Iamblichus's account (also see Zhmud 2006: 132, where he refers to Hippocrates of Chios, Archytas, and Eudoxus as "typical" μαθηματικοί).

13. Riedweg's account (2005. 106–108) is probably the best synthetic account outside of Burkert (1972), although we should recognize the care with which Burnyeat (2005a) examined the philosophical context in Aristotle (without analysis of the political aspects of the reported schism). Burnyeat thus leads the way for my study.

14. As I will suggest below, Iamblichus goes on to quote the work directly.

> Δύο δ' ἐστὶ τῆς Ἰταλικῆς φιλοσοφίας εἴδη, καλουμένης δὲ Πυθαγορικῆς. δύο γὰρ ἦν γένη καὶ τῶν μεταχειριζομένων αὐτήν, οἱ μὲν ἀκουσματικοί, οἱ δὲ μαθηματικοί. τούτων δὲ οἱ μὲν ἀκουσματικοὶ ὡμολογοῦντο Πυθαγόρειοι εἶναι ὑπὸ τῶν ἑτέρων, τοὺς δὲ μαθηματικοὺς οὗτοι οὐχ ὡμολόγουν, οὔτε τὴν πραγματείαν αὐτῶν εἶναι Πυθαγόρου, ἀλλὰ Ἱππάσου.
>
> (Iamblichus, *On the General Mathematical Science* 25, 76.16–22)

Now Burkert synthesizes the material derived from Aristotle's works on the Pythagoreans and preserved by Iamblichus[15] in order to demonstrate two significant points: first, that *all* followers of Pythagoras were adherents of the *acusmata*, also called *symbola*,[16] a set of orally transmitted sayings passed down from Pythagorean teacher to student in the period of silence that apparently attended the first five years of their educational curriculum, and second, that what distinguished the ascetic acousmatic Pythagoreans (ἀκουσματικοί) from the progressive mathematical Pythagoreans (μαθηματικοί) was each group's unique philosophical and political *pragmateia*:

> Aristotle recognizes among the Pythagoreans a twofold πραγματεία: on the one hand, the Πυθαγορικοὶ μῦθοι, metempsychosis, the Pythagoras legend, and the *acusmata*, and on the other a philosophy of number connected with mathematics, astronomy, and music, which he never tries to trace back to Pythagoras himself and whose chronology he leaves in abeyance.[17]

Furthermore, Burkert argues that Aristotle categorized the Pythagorean *acusmata* according to whether or not they answered these three questions: τί ἔστι (what is?), τί μάλιστα (what is to the greatest degree?), and τί πρακτέον (what is to be done?).[18] While the implications of this fascinating tripartite categorization both for Aristotelian philosophy and for Pythagoreanism could extend far

15. Zhmud's arguments (2012: 174) that suggest Clement of Alexandria as the *source* for the division into ἀκουσματικοί and μαθηματικοί are not decisive. For one thing, it remains for Zhmud to explain the philosophical language of the passage quoted by Iamblichus. See below in chapters 1 and 2 for my alternative treatment of the evidence.

16. See Zhmud 2012: 173 with n. 16.

17. Burkert 1972: 197.

18. See Burkert 1972: 167–169, with Iambl. *VP* 82, 47.11–13, and Delatte 1915: 274–307. Burkert rightly reminds us that these "orally transmitted maxims and sayings" were also called *symbola*. Recently, Struck (2004: 96–110) has attempted a comprehensive study on symbolic or enigmatic communication in antiquity, although his book also does not treat the third kind of *acusma*.

beyond this study, throughout this chapter I focus chiefly on the third classification, namely on those things that fall under the category τί πρακτέον.

Burkert explicates those *acusmata* that fall under the category "what is to be done" by focusing, almost entirely, on ethical imperatives and ritual activity.[19] He demonstrates their significance for the establishment of a Pythagorean way of life as an "amazing, inextricable tangle of religious and rational ethics."[20] This is a valuable approach to understanding one important aspect of the philosophical lifestyle ascribed to the Pythagoreans, because it reveals the religious semantics of *pragmateia*. Burkert's study also reflects its own Aristotelian intellectual lineage since, as Iamblichus argues (in the Aristotelian analysis of the "what is to be done" injunctions that follows on their listing), what is divine (τὸ θεῖον) is the first principle and origin (ἀρχή).[21] But I suspect that there is more to Aristotle's classification of the two Pythagorean *pragmateiai* than Burkert discusses. For Aristotle, as for Archytas, the term *pragmateia* was chiefly associated with philosophical methodology, and not only with theology, although the latter is implicated in the former. Can we gain some traction on the philosophical activities of the Pythagoreans by examining more closely this implication of theology in philosophical activity? One passage from Iamblichus's work *On the Pythagorean Life*, probably derived from a Peripatetic account of Pythagoreanism, helps to show the way:

> All such *acusmata*, however, that define what is to be done or what is not to be done [περὶ τοῦ πράττειν ἢ μὴ πράττειν], are directed toward the divine [ἐστόχασται πρὸς τὸ θεῖον], and this is a first principle [ἀρχή], and their whole way of life is arranged with a view to following God [ὁ βίος ἅπας συντέτακται πρὸς τὸ ἀκολουθεῖν τῷ θεῷ], and this is the rationale [λόγος] of their philosophy.
>
> (Iamblichus, *On the Pythagorean Way of Life* 86–87, 50.18–21; translation after Dillon and Hershbell 1991)

One of the great challenges of this passage is to extract what, if anything, traces back to the fourth century BCE. We may never be absolutely certain.[22] The reference

19. Burkert 1972: 174–192. Similarly followed by Kahn (2001: 9–10) and Riedweg (2005: 63–67).

20. Burkert 1972: 185.

21. Iambl. *VP* 86, 50.18–19. Note that Aristotle makes a similar claim at *Metaph.* 1.2, 983a6–11, on which see Nightingale 2004. 236–237.

22. Zhmud (2012: 189) thinks this passage derives from Nicomachus, but I think it is an overstatement to describe the differences between the various passages of *VP* 81, 87–89, and 82–86 as "self-evident," especially since, as Zhmud himself admits (p. 191), there are "clear signs of editorial emendations by Iamblichus." A related problem here is the grammar

to the *acusmata*—especially those that deal with "what one is to do" injunctions—*sounds* Aristotelian, as does the ascription of divinity to the first principle. A likely source for this part of the text, as I argue in chapter 2, is Aristoxenus of Tarentum, who speculated about the Pythagorean first principle in related ways in his *Pythagorean Precepts*.[23] In the passage that immediately precedes this one, however, the attempt to define a "first principle" (ἀρχή) and a "reason" or "rationale" (λόγος) for the Pythagorean philosophy as related to the first principle is characteristic of Aristotle's method of describing and critiquing earlier philosophical systems. We might, for example, recall the beginning of the *Nicomachean Ethics* (1.4, 1095a30–b14), where Aristotle questions whether it is better to employ arguments (λόγοι) that *derive from* first principles (ἀπὸ τῶν ἀρχῶν) or those that *lead to* first principles (ἐπὶ τὰς ἀρχάς). In this digression, Aristotle appears to distinguish his own philosophical method from Plato's by arguing that we should begin from what is already known and familiar to us, namely, the "what is" or "fact" (τὸ ὅτι), which he also calls a "first principle" (ἀρχή).[24] With regard to first principles, Aristotle's approach here stands in contrast to the approach attributed to the Pythagoreans in Iamblichus's work *On the Pythagorean Way of Life* 86–87, which attributes to Pythagoreans the sorts of λόγοι that reduce to the first principle, namely the divine.[25]

But which Pythagoreans, acousmatics or mathematicians, was Iamblichus describing in this passage? Or was he talking about the *pragmateia* of all the Pythagoreans? There is no standard scholarly position on this question, in part because scholars have been unclear about which sections derive from the Peripatetic source, or how much Iamblichus has doctored the text.[26] It is likely,

of "aiming" (ἐστόχασται). While it is the case that Aristotle speaks of "aiming at" objects such as a "good" (*Pol.* 1.1, 1252a4), "pleasure" (*Metaph.* 6.2, 1027a3) or "the mean" (*EN* 2.9, 1109a30–2), the object at which one aims is always in the genitive case, whereas in Iamblichus it is in the πρὸς + accusative phrase.

23. See chapter 2, section entitled "Pythagoreanism and the Axiology of What Is 'Honorable.'"

24. ἀρχὴ γὰρ τὸ ὅτι· καὶ εἰ τοῦτο φαίνοιτο ἀρχούντως, οὐδὲν προσδεήσει τοῦ διότι. On the relationship between the "fact" and the "why," see Burnyeat 1981: 118 and, more recently, Zhmud 2006: 136.

25. In this way, the Aristotelian passage preserved in *VP* 86–87 may have formed the basis for (or referred to the same system described by) Aristoxenus's account of the *Pythagorean Precepts*, especially F 33 Wehrli (= Iambl. *VP* 174–176, 97.23–98.24) and F 34 Wehrli (= Stob. *Ecl.* 4.25.45), which describe the ontological stratification of being for the "Pythagoreans." See Huffman 2006: 112 and 2008: 107–108. Theophrastus (*Metaph.* 11a26–b12) also speaks of Plato and the "Pythagoreans" as reducing to the first principles, on which see Horky: forthcoming.

26. See Burkert 1972: 196 n. 17.

I suggest, that the reference is to the Pythagoreans in general, and not to a particular group, in this passage. While it is true that the distinction between acousmatic and mathematical Pythagoreans immediately precedes this passage in Iamblichus's text, there are three reasons for interpreting this passage as referring to Pythagoreans more generally. First, Iamblichus separates a long passage where he discusses the distinctions between two groups of Pythagoreans (*On the Pythagorean Way of Life* 81–86, 46.23–50.17) by a poignant "however" (μέντοι), suggesting that he has completed discussion of the split between two groups of Pythagoreans.[27] Second, there is nothing specific to suggest that we should identify the system of religious order described as acousmatic or mathematical: this is unsurprising, since it is generally agreed that the mathematical Pythagoreans accepted the religious and ethical precepts of the acousmatic Pythagoreans.[28] Finally, when Iamblichus returns to discussing the *acusmata* later in the treatise (*On the Pythagorean Way of Life* 137, 77.13–19), he repeats this passage almost verbatim and describes it as illustrating the principles of religious worship of the gods as attributed to Pythagoras and to his followers (Πυθαγόρας τε καὶ οἱ ἀπ'αὐτοῦ ἄνδρες). Thus, the broader description of the *pragmateia* of the Pythagoreans (as formulated by Iamblichus in his work *On the Pythagorean Way of Life* 86–87) focuses on two important aspects that I will continue to discuss in this study: the hierarchy of the cosmos, which one honors by understanding that the divine is the first principle that must be pursued in order to attain the good; and the hierarchy of a political organization, which is analogous to the cosmic hierarchy. In this way, when Iamblichus's Peripatetic source characterized the universal Pythagorean *pragmateia*, he seems to have exploited both the religious and political senses of the term ἀρχή.[29]

Close attention to *philosophical methodologies*, however, might give us a better insight into the rationales that distinguished the *pragmateiai* of the different Pythagoreans. When he describes the rationale (λόγος) for the maxims that

27. Iamblichus synthesizes the descriptions of the two groups: the first group, the ἀκουσματικοί, are said initially (*VP* 82, 47.4–6) to practice a philosophy "without demonstration and without argument" (ἀναπόδεικτα καὶ ἄνευ λόγου) and are later (*VP* 86, 50.9–12) associated with those who undertake philosophical activity that is properly "Pythagorean" (Πυθαγορικαί); and the μαθηματικοί, who offer up "probable reasons" (εἰκοτολογίαι), are called "some [others] from outside" (ἔνιοι ἔξωθεν). As I will show, I believe the distinction given earlier between ἀκουσματικοί and μαθηματικοί to be Aristotelian, whereas I suggest that the later differentiation between those "inside" and "outside" the school may derive from Timaeus of Tauromenium. See chapter 3.

28. See Huffman 2010, Riedweg 2005: 106–107, and Kahn 2001: 15.

29. Aristoxenus is explicit in exploiting both meanings by reference to the Pythagoreans and is the likely source here. See chapter 2, section entitled "Pythagoreanism and the Axiology of What Is 'Honorable.'"

answer the question τί πρακτέον, Iamblichus (*VP* 86, 50.6–13) distinguishes the use of rationales by the more conservative Pythagoreans from the use by those people whose philosophical activities he claims are "non-Pythagorean" (οὐκ εἰσὶ Πυθαγορικαί) and who are also called "outsiders" (ἔξωθεν). Are those figures designated "outsiders" the same as the mathematical Pythagoreans?

The evidence concerning the "esoteric" and "exoteric" Pythagoreans in Iamblichus's work is ambivalent, but it is not likely, I suggest, that Aristotle understood the division along insider and outsider lines.[30] Rather, as I will show in chapter 3, the source for the passages that distinguish "exoteric" from "esoteric" Pythagoreans in Iamblichus's work *On the Pythagorean Way of Life* appears to be the late fourth-/early third-century BCE Western Greek historian Timaeus of Tauromenium, who posited a division between those Pythagoreans who were more advanced in their learning (inside) and those who did not advance beyond a certain level (outside).[31] Even so, the source for this part of *On the Pythagorean Way of Life* 86 still evinces a division along philosophical grounds. These "exoteric" Pythagoreans differ from the "esoteric" Pythagoreans specifically because they "attempt to attach a likely rationale/account" (πειρωμένων προσάπτειν εἰκότα λόγον) to the ethical injunctions that constitute the Pythagorean *acusmata*.[32] The "likely account" (εἰκοτολογία) that Iamblichus's source attributes to those people who are "non-Pythagoreans" or "exoteric" in this passage represents a more sophisticated approach to wisdom traditions such as those of Pythagoras or the Seven Sages, but it is not "mathematical" in the strong sense, at least if we are to judge by the examples given. The sorts of "likely account" given by the "exoteric" Pythagoreans are focused on practical—indeed, even political—reasoning in a way not unlike the εἰκὼς λόγος given by Timaeus of Epizephyrian Locri in Plato's *Timaeus* and developed in some of Aristotle's works, including the *Politics*.[33] According to Iamblichus's source, those who were described as "exoteric" Pythagoreans exhibited different types of *logos*, including cultural-historical explanation ("one should not break bread" *because*, in the past, people used to come together in order to eat a single loaf of bread, as foreigners do) and normative-religious ("one should not break bread" *because* one ought not to establish the sort of omen that occurs at the beginning of the meal by means of breaking and crushing bread).[34] Such examples suggest that the "exoteric" Pythagoreans whose *pragmateia* involved cultural-historical or normative-religious types of *logos* appealed to fifth century BCE sorts of explanation, such as those we find in the writings of Herodotus or the writers of the Hippocratic Corpus.[35] They appear, in this account, to resemble more the Pythagorists of Middle Comedy who know how to make clever arguments by using various fallacious devices (ἐπισοφιζομένων), or even the sorts of Presocratics whose speculations formed the basis for the character of Socrates in Aristophanes's *Clouds*, than highly regarded practitioners of wisdom. Still, accusations of illegitimate claims to wisdom are as old as Pythagoras himself, and they were of interest to Timaeus of Tauromenium: our source for Heraclitus's slander of Pythagoras, in which he refers to Pythagoras as a "prince of lies" (κοπίδων ἀρχηγός), is Timaeus himself.[36]

30. Of course, Aristotle himself referred to some of his writings as "exoteric" (ἐξωτερικοὶ λόγοι), which, at *EE* 1.8, 1217b22 he sets in contrast to those writings that he calls "philosophical" (οἱ κατὰ φιλοσοφίαν λόγοι). Much has been said about this distinction, and little is agreed on (for two divergent recent accounts, see Gerson 2005: 47–76 and Zanatta 2008: 26–35). What is of value for this study is that the version of the "exoteric"/"esoteric" division found in Iamblichus's works is *never explicitly* drawn by Aristotle and, therefore, probably owes its origins to someone else. For a useful study of the relationship between the terms "exoteric/esoteric" and "acousmatic/mathematical" Pythagorean, see von Fritz 1960: 8–10.

31. In chapter 3, I explore at much greater length Timaeus of Tauromenium's criticisms of Aristotle's history of the Pythagoreans.

32. The term εἰκὼς λόγος, which is technical, receives a great number of conflicting treatments in antiquity. In Plato's *Timaeus* (30b8), it refers to the "likely story" that cannot, on Morgan's reading (2000: 275), be verifiable by appeal to empirical knowledge. It is interesting to note that Ps.-Archytas's *On Intelligence and Perception* (F 1 Thesleff = Stob. 1.41.5) refers to εἰκοτολογίαι in reference to political treatises, namely things that deal with "affairs" (πράξιας).

33. See Burnyeat 2005b, who emphasizes the reasonableness or appropriateness (the "ought": δεῖ) that constitutes the goal to which the practitioner of the εἰκὼς λόγος aims. I consider Plato's *Timaeus* to be an "exoteric" Platonic dialogue, in the sense that it makes public *and explains* what might otherwise be considered "unspeakable" ideas in a fourth-century BCE context to an indistinguished audience.

34. It is worth noting that the information preserved here is almost exactly the same as that attributed by Diogenes Laertius to Aristotle's work *On the Pythagoreans* (F 195 Rose = D.L. 8.33–35). It is possible, then, that Iamblichus was looking at Aristotle's text while recording this information or, for that matter, that the historian Timaeus of Tauromenium had access to Aristotle's text while drawing up his own list of the *acusmata* (on which see chapter 3).

35. For Herodotean ἱστορία and its contexts, see Lateiner 1989: 15–17 and Thomas 2000: 21–27; for Presocratic and Hippocratic ἱστορία see Schiefsky 2005: 19–35; more generally, for philosophically related uses of ἱστορία before Plato, see Riedweg 2005: 94–95 and Darbo-Peschanski 2007.

36 FGrHist 566 F 132 (see DK 22 B 81). The term ἐπισοφίζομαι occurs in Iamblichus and in post-Iamblichean texts, but it is also attested in the Hippocratic corpus (*Art.* 14) with reference to clever doctors who demonstrate their cleverness by attaching a piece of lead to a fractured bone in order to stabilize it. See Burkert 1972: 174 with n. 64 and 200. I would add, however, that such "cleverness" is attached to the Tarentine Pythagoreans whose rhetorical *logoi* are satirized in two plays, both entitled *The Tarentines*, written by the

So *On the Pythagorean Way of Life* 86–87 presents us with a paradigmatic case of the problems involved in sorting out the sources of Iamblichus's information concerning the classification of the Pythagoreans: not only must we deal with the terminology of *at least* two different historians (Aristotle and possibly Timaeus, not to speak of Aristoxenus or Nicomachus), we have to be sensitive to how Iamblichus might have confused the accounts. Despite this hindrance, we can gain some traction on the question of the philosophical activities of the various Pythagoreans as Aristotle figured them by appeal to a passage, preserved by Iamblichus fortunately with some direct quotation:

> (A) There are two types of the Italian, also called the Pythagorean, philosophy. For there were also two kinds of people who treated it: the acousmatics and the mathematicians. Of these two, the acousmatics were recognized to be Pythagoreans by the others [the mathematicians], but they did not recognize the mathematicians [as Pythagoreans],[37] nor did they think that the *pragmateia* [of the mathematicians] derived from Pythagoras, but rather that it derived from Hippasus. Some say that Hippasus was from Croton, while others say from Metapontum.[38] And, of the Pythagoreans, those who concern themselves with the sciences [οἱ περὶ τὰ μαθήματα][39] recognize that the others [i.e. the acousmatics] are Pythagoreans, and they declare that they themselves are even more [Pythagorean], and that the things they say [ἃ λέγουσιν] are true. And they[40] say that the reason [αἰτία] for such a disagreement is this:
>
> (B) "Pythagoras came from Ionia, more precisely from Samos, at the time of the tyranny of Polycrates, when Italy was at its height, and the first men of the city-states became his associates. The older of these [men] he addressed in a simple style, since they, who had little leisure on account of their being occupied in political affairs, had trouble when he conversed with them in terms of sciences [μαθήματα] and demonstrations [ἀποδείξεις]. He thought that they would fare no worse if they knew *what* they ought to do [εἰδότας τί δεῖ πράττειν], even if they lacked the explanation [ἄνευ τῆς αἰτίας] for it, just as people under medical care fare no worse when they do not additionally hear *the reason why* they ought to do [διὰ τὶ πρακτέον] each thing in their treatment. The younger of these [men],

fourth-century BCE comedians Alexis of Thurii (F 223 K.-A.: Πυθαγορισμοὶ καὶ λόγοι / λεπτοὶ διεσμιλευμέναι τε φροντίδες / τρέφουσ' ἐκείνους) and Cratinus the Younger (F 7 K.-A.: ἔθος ἐστὶν αὐτοῖς . . . διαπειρώμενον / τῆς τῶν λόγων ῥωμῆς ταράττειν καὶ κυκᾶν / τοῖς ἀντιθέτοις, τοῖς πέρασι, τοῖς παρισλώμασιν, / τοῖς ἀποπλάνοις, τοῖς μεγέθεσιν, νουβιστικῶς). We can thus posit a popular tradition, not necessarily derived from Aristotle, that attributes sophisms of a rhetorical sort to the Tarentine Pythagoreans. Note, too, that Cratinus employs terms both rhetorical and mathematical, such as πέρας and μέγεθος, translated by Edmonds as "end" and "sublimity." The former is attested in a rhetorical sense in the Aristotelian *Rhetoric to Alexander* (32, 1439a38), where it is described as the conclusion that rounds off an exhortation. The latter appears in Aristotle's *Rhetoric* (3.9, 1409a36), with reference to periodic sentences that can be measured, as well as in Dionysius of Halicarnassus (*Comp.* 17), as "sublimity." It is difficult to know precisely what Cratinus the Younger intended their meaning to be.

37. Iamblichus elsewhere (*VP* 87, 51.7–12), in a passage that is attached to the same one given in *DCM*, attributes to a certain acousmatic Pythagorean "Hippomedon" the claim that Pythagoras originally gave demonstrations of the precepts, but that, due to the laziness of those who passed them down, ultimately only the precepts remained. Unfortunately, it is difficult to confirm this information, since (1) there are textual problems here (see Deubner's text); (2) we know almost nothing else about this Hippomedon; (3) it is possible that Iamblichus has confused "acousmatic" with "mathematical" Pythagorean here, as he did earlier at *VP* 81, 46.26–47.3 (see Burkert 1972: 198 n. 8).

38. It is not clear to me whether this sentence is Iamblichus's insertion or original with his source, who is probably Aristotle.

39. The term μαθήματα is extremely difficult to translate, and no single translation will do justice. Alternatives include "learning" or "mathematics," but I think Burkert (1972: 195 and 207 n. 80) is correct in defining this term as the branches of learning the Greeks called arithmetic, geometry, astronomy, and music. We should note that Archytas specifically refers to his predecessors as τοὶ περὶ τὰ μαθήματα (Archytas F 1 Huffman) and attributes to them innovations in scientific method, especially concerning numbers, geometry, music, and the motions of the stars. In chapter 6, I argue that Archytas is referring chiefly to Hippasus when he speaks of his predecessors.

40. Who is the subject of this φασίν? Stylistically, there is a minor change of tune from the previous section, which had focused on whether or not the acousmatics or mathematicians "recognized" one another (various forms of ὁμολογεῖν), where a distinction is drawn between the acousmatics who "did not recognize" (in the imperfect tense) the mathematicians as Pythagoreans, and the mathematicians who "recognize" (in the present tense) the acousmatics as Pythagoreans. The appearance of the phrase οἱ περὶ τὰ μαθήματα in that earlier section suggests the possibility, indeed, that the information might derive ultimately from Archytas (see the previous note). And, as I argue in chapter 3, Archytas and other mathematical Pythagoreans wrote about their predecessors. But the appearance of concern with "reason" or "cause" (αἰτία), which is followed up in the portion that seems to be quoted directly (B), which focuses on causation, suggests that someone who formulated a philosophical engagement with causation is responsible for the information that follows. From what remains of Archytas's fragments, there is no obvious interest in causation as such; but Eudemus's account of Archytas's physics (A 23 Huffman) suggests that he did believe that inequality and unevenness were causes of motion. And he was concerned with demonstration as well (F 4 Huffman). Still, we cannot be sure that Eudemus has not mapped Peripatetic terminology onto Archytas's ideas about physics. The most obvious candidate for the subject of this φασίν, then, remains Aristotle, as Burkert originally argued (1972: 457), and as Burnyeat has confirmed (2005a: 40–43). Possibly this material derives from one of Aristotle's works on Archytas. Thanks to Monte Johnson for pressing me to clarify my position on this issue.

however, who had the ability to endure the education, he conversed with in terms of demonstrations and sciences. So, then, these men [i.e. the mathematicians] are descended from the latter group, as are the others [i.e. the acousmatics] from the former group."

(C) And concerning Hippasus, they say that while he was one of the Pythagoreans, he was drowned at sea for committing heresy, on account of being the first to publish, in written form [διὰ τὸ ἐξενεγκεῖν καὶ γράψασθαι],[41] the sphere, which was constructed from twelve pentagons. He acquired fame for making his discovery, but all discoveries were really from "that man" [as they called Pythagoras; they do not call him by name][42] . . . well, then, such are basically the characteristic differences between each philosophical system and its particular science.[43]

(Iamblichus, *On the General Mathematical Science* 25, 76.16–78.8)

This passage of Iamblichus, which is the central evidence for Aristotle's version of the factionalization of the Pythagorean brotherhood,[44] further supports my claim that what primarily distinguished the acousmatic and mathematical Pythagoreans was the object of their philosophical inquiry and treatment of that object (*pragmateia*). The passage can be divided into three sections: (A), which, while not obviously direct quotation, is nonetheless derived, in great part (if not wholly), from Aristotle's lost writings on the Pythagoreans; (B), which is apparently direct quotation from Aristotle; and (C), which is also likely to be derived from Aristotle.[45] In the section apparently quoted directly from one of Aristotle's lost works on the Pythagoreans (B), what distinguishes the acousmatic from the mathematical Pythagoreans is *type of knowledge*: the

41. For a more precise analysis of what this phrase means in context, see chapter 2, section entitled "Aristotle on Hippasus of Metapontum."

42. There is likely to be an interpolation here, which originally came from the *History of Arithmetic* of Eudemus of Rhodes. See Zhmud 2006: 187.

43. A very similar version found at Iambl. *VP* 87–89, 51.12–52.14, but—notwithstanding the confusion of acousmatic and mathematical, discussed in note 37, and the interpolation probably from Eudemus—there Iamblichus substitutes the "followers of Pythagoras" (τῶν ἀνδρῶν τῶν ἀκροωμένων) for "sciences" (τῶν μαθημάτων) and "we have ascertained" (παρειλήφαμεν) for "such are the characteristic differences" (τοιαῦτά ἐστι τὰ συμβεβηκότα). The presence of the Aristotelian term συμβεβηκότα in *DCM* probably indicates the more original text.

44. We can compare this account with that given by Iamblichus at *VP* 247, 132.18–21, whose provenance is unclear (possibly Nicomachus).

45. These divisions accord with the switch to indirect discourse and return to direct discourse. I will discuss (C) more extensively in chapters 2 and 3.

acousmatic Pythagoreans only have knowledge of the fact of "what one is to do" (τί δεῖ πράττειν), but the mathematical Pythagoreans, whose understanding is more advanced, have knowledge of the "reason why they are to do" (διὰ τὶ πρακτέον) what they should do.[46] This methodological distinction between "fact" (ὅτι) and "reason why" (διὰ τί) is originally Aristotelian, and it thus corroborates my suggestion that passage (B), and possibly the contingent passages (A) and (C), derive from Aristotle.[47] Indeed, the distinction between the "fact" (ὅτι) and the "reason why" (διότι) is central in Aristotle's controversial description of the knowledge of mathematicians in the *Posterior Analytics*:

The reason why [τὸ διότι] is superior to[48] the fact [διαφέρει τοῦ ὅτι] in another way, in that each is studied by means of a different science. Such is the case with things that are related to one another in such a way that one is subordinate to the other, e.g. optics to geometry, mechanics to stereometry, harmonics to arithmetic, and star-gazing to astronomy. Some of these sciences bear almost the same name, e.g. mathematical and nautical astronomy are called "astronomy," and mathematical and acoustical harmonics are called "harmonics." In these cases it is for those who concern themselves with perception to have knowledge of the facts [τὸ ὅτι εἰδέναι], whereas it is for the mathematicians to have knowledge of the reason why [τὸ διότι εἰδέναι]. For the latter grasp[49] demonstrations of the causes [τῶν αἰτίων τὰς ἀποδείξεις], and they often do not know the facts [τὸ ὅτι], just as people who study the universal often do not know some of the particular instances *for lack of observing them*.[50] The objects of their study are the sort that, although they are something different in substance, make use of forms [κέχρηται τοῖς εἴδησιν]. For mathematics is concerned with forms; its objects are not said of a particular substrate.

(Aristotle, *Posterior Analytics* 1.13, 78b34–79a8)

46. The distinction is also identified by Iamblichus at *VP* 82, 47.4–10.

47. Some scholars (e.g. Zhmud 2012: 186 and, following him, Afonasin 2012: 31 n. 75) have speculated that this whole passage is chiefly derived from Nicomachus; but there is simply no evidence of Nicomachus adopting the Aristotelian differentiation between subordinate and superordinate sciences, which, I argue, underlies the differentiation of types of Pythagorean philosophical activity in this passage. Nor is there any extant evidence adduced by Zhmud to show that Nicomachus himself was concerned with the epistemic status of demonstration or proof.

48. Or, possibly, "differs from." But the language of subordination here suggests that Aristotle was using the common Greek idiom διαφέρει + genitive to mean "is superior to" or "excels." See *LSJ* s.v. διαφέρω 3.4.

49. Translating ἔχουσι literally, but the sense might be something like "able [to make]."

50. My italics.

This description of the so-called subordinate sciences develops a useful analogue for how acousmatic Pythagoreans differ from the mathematicians. The philosophy of the acousmatics, which is described by Iamblichus (*On the Pythagorean Way of Life* 82, 47.4–6) as consisting of "*acusmata* undemonstrated [ἀκούσματα ἀναπόδεικτα] and without argument [ἄνευ λόγου]," focuses on knowledge of "what" to do (ὅτι πρακτέον), not the reasons "why" to do it. By contrast, the mathematical Pythagoreans obtain the same characteristics as the mathematicians described in the *Posterior Analytics*, who have knowledge of the "reason why" and are able to grasp and produce "demonstrations" of the causes of the objects of their study. Aristotle's characterization of mathematicians as people who make use of demonstrations in their philosophical *pragmateia* parallels that of the mathematical Pythagoreans in the Aristotelian passage (B) quoted in *On the General Mathematical Science* 25, 77.4–18, although, importantly, there is no reference to Aristotle's peculiar understanding of mathematical "forms" or "substance" in Iamblichus's text. If the work quoted from was composed very early in Aristotle's career, before he undertook new approaches to ontology in the *Categories*, it would not be surprising that we do not hear about such problems. Be that as it may, my analysis of the passages that preserve some material from Aristotle's lost works on the Pythagoreans in Iamblichus's work *On the Pythagorean Way of Life* and *On the General Mathematical Science* reveals strong links to the differentiation of the two types of science in the *Posterior Analytics*, which leads to the supposition that Aristotle saw the main differentiating factor between the acousmatic and the mathematical Pythagoreans as demonstration.

ON THE "SO-CALLED" AND MATHEMATICAL PYTHAGOREANS

The establishment of sections (A) and (B) from Iamblichus's work *On the General Mathematical Science* 25, 76.16–77.18 as derived generally from Aristotle's writings on the Pythagoreans is very important for our understanding of mathematical Pythagoreanism, as Aristotle constructed it, because it corroborates a claim that has often been suggested but never explicitly argued for by scholars:[51] that the "so-called" Pythagoreans (οἱ καλούμενοι Πυθαγόρειοι) to whom Aristotle refers in *Metaphysics* A (1.5, 985b23 and 1.8, 989b29), *On the Heavens*

51. See Burkert 1972: 30 with nn. 8–9 and 51–52, who is followed by Huffman (1993: 31–35). Huffman's suggestion that others who might be "so-called" Pythagoreans would include Hippasus, Lysus, and Eurytus is plausible, although I doubt that those who proposed the theory of *sustoicheia* would be included. The most extensive analysis of this problem was undertaken by Timpanaro Cardini (1964: 6–19), but she concludes erroneously, I would argue, that there is no distinction between the various types of Pythagoreans named in Aristotle's *Metaphysics*.

(2.1, 284b7 and 2.13, 293a20–21), and *Meteorology* (1.6, 342b30 and 1.8, 345a14) are, indeed, one and the same with the mathematical Pythagoreans described in the lost works on the Pythagoreans.[52] Given my new approach to thinking about the *pragmateia* of the mathematical Pythagoreans, that is, the object of their philosophical investigations and their particular treatment of that object, it is worth considering whether there might be parallels to draw with the "so-called" Pythagoreans in those texts.

Let us examine a famous passage from the first book of Aristotle's *Metaphysics*, which one might assume (with Jaeger, Ross, and Owens)[53] to have been written rather early in Aristotle's career, when he was still under Academic influence:

> The "so-called" Pythagoreans employ first principles and elements [ταῖς ἀρχαῖς καὶ στοιχείοις χρῶνται] more abstrusely[54] than some of the physicists. The reason is that they took their first principles from non-perceptible objects: for the objects of mathematics [τὰ μαθηματικὰ τῶν ὄντων],[55] apart from those that concern astronomy, belong to the class of things lacking in motion. And yet they discuss and wholly make the object of their philosophical inquiry [πραγματεύονται] nature. For they generate heaven, and they observe what happens concerning its parts, attributes, and functions, and they lavish these things with first principles and causes, and as such they are in agreement with the other natural scientists that what actually exists is what is perceived and that "so-called" heaven contains it. But, as we mentioned, the causes and the first principles, which they say are sufficient to rise up above the horizon [ἐπαναβῆναι][56] to the higher parts

52. I will deal primarily with the passages in *Metaphysics* A, for the sake of their strong connections with the fragments of Aristotle's lost works on the Pythagoreans. It should be noted that the term "so-called" is not particularly innovative for Aristotle, given that skepticism concerning people who called themselves after Pythagoras can be detected in the writings of Isocrates and Antisthenes. See chapter 2.

53. See Owens 1951: 85–89; Jaeger 1948: 171–176; Ross 1924, vol. 1: xv.

54. ἐκτοπωτέρως, following Asclepius's commentary (*in Metaph.* p. 65.29–35 Kroll) and the most recent edition of Primavesi 2012.

55. As I translate this very tricky phrase. Literally, it means something closer to "the mathematicals among the things," which coordinates in potentially interesting ways with Philolaus's (F 6 Huffman) phrase "the being of things" (ἁ ἐστὼ τῶν πραγμάτων). See below in the section entitled "Mathematical Pythagoreanism and the 'Objects of Mathematics.'"

56. This translation is preferable to Tredennick's "capable of application to the remoter class of realities" or Ross's "sufficient to act as steps even up to the higher realms of reality," neither of which accounts for the technical language of astronomy reported here. In a passage of the

of reality, are better suited even for these than for arguments concerning nature. Nevertheless, they say nothing about how there will be motion, if the only things premised are limit and limitless, and odd and even, nor about how there can be generation and destruction, nor the activities of objects that move through the heavens, without motion and change.

Further, if someone were to grant to them that spatial magnitude derives from these things, or if this were to have been demonstrated by them [δειχθείη τοῦτο], still how will some bodies be light and others heavy? For, given what they assume and maintain, they are speaking no more about mathematical bodies than about perceptible bodies. Hence they have said nothing whatsoever about fire or earth or any other bodies of this sort, since, in my opinion, nothing they say is peculiar to perceptible bodies.

Moreover, how is one to understand that both the attributes of number and number itself are the causes of things that exist and come to be throughout the heavens—both from the beginning and now—and that there is no other number than this number out of which the cosmos is composed? For, whenever they place opinion and opportunity in such and such a region, and injustice and separation or mixture a bit higher or lower, and they make a demonstration on the grounds that [ἀπόδειξιν λέγωσιν ὅτι] each of these is a number—but there already happens to be a plurality of magnitudes composed [of numbers] in that place, because the attributes correspond to each of these places—is, then, the number in heaven, which one is supposed to understand as each [of these abstractions], the same [as the one in the lower region], or is it a different number?

(Aristotle, *Metaphysics*, 1.8, 989b29–990a29)

Obviously, there is a great deal to unpack in this extended discussion of the "so-called" Pythagoreans and their relationship to the mathematical Pythagoreans. I would like to highlight just a few aspects of Aristotle's argument that are relevant to this analysis.

Aristotle seeks a technical language in order to respond to what he takes as the fundamental aspects of the "so-called" Pythagorean philosophical system. He points out category confusion in "so-called" Pythagorean philosophy: while their first principles are all derived from the mathematical (i.e. non-perceptible)

Meteorology (1.6, 342b30–35), Aristotle describes how the "some" of "so-called" Pythagoreans believe that Mercury is, like comets, one of the Planets that "does not rise far above the horizon" (τὸ μικρὸν ἐπαναβαίνειν), and therefore its appearances are invisible, as it is seen in long intervals.

sciences, the objects of their philosophical inquiry (i.e. their *pragmateia*) are things that have been generated and possess motion, namely phenomena. This would be unsurprising, especially if the "so-called" Pythagoreans made the object of their investigations the motions of the heavens, the superior science of which would be, according to the *Posterior Analytics*, stereometry. But the example he gives involves number: how can number, which is a non-perceptible entity, both (1) be superordinate (i.e. a "first principle") and reside in the highest part of reality and (2) be identical with something in a lower substrate, like opinion or opportunity? In accordance with Aristotle's establishment of the proper objects to the various sciences in the *Posterior Analytics*, this "so-called" Pythagorean approach represents a confounding of the sciences that deal with the "reason why" (τὸ διότι) and the sciences that deal with the "fact" (τὸ ὅτι). That is to say, it leads to confusion about what kind of science the "so-called" Pythagoreans practice, since they employ the principles of theoretical mathematics in order to explain natural phenomena.

Regardless of Aristotle's criticisms of the "so-called" Pythagoreans, we can see that they were thought to have undertaken demonstrations of some sort, which suggests to us that they are the same as the mathematical Pythagoreans Aristotle described in his lost works on the Pythagoreans. If we are to trust Aristotle's evidence here, then the mathematical Pythagoreans described in *Metaphysics* A may have provided at least two types of demonstrations: (1) that all entities that are derived from number are themselves numbers, on the grounds that all entities possess the attributes of number, and, possibly, (2) that spatial magnitude is derived from their first principles, namely the objects of mathematics.[57] Other demonstrations ascribed to the "so-called" Pythagoreans in Aristotle's works are suggestive, if incomplete, evidence for the explanatory methods employed by these philosophers.[58] In this way, they are distinguished from the acousmatic Pythagoreans, whose philosophical *pragmateia* is said to have been focused uniquely on the "fact" (τὸ ὅτι), that is, that which is particular, mutable, perceptible, and known through experience alone.[59] However,

57. That Aristotle mentions the proof concerning magnitude suggests it is possible that someone could have (or did) try to demonstrate this.

58. Among those that I will not discuss further: at *Cael.* 2.2, 284b6–8, they have a *logos*—it is unclear how it is demonstrated—that argues that there is a right and left side in heaven; at *Mete.* 1.8, 345a14–19, we hear of two kinds of arguments attributed to "so-called" Pythagoreans: the first is mythological (the Milky Way is a path on the grounds that it is the path of one of the stars that fell at the time of the fall of Phaethon), and the second is based on stereometric speculation and natural science (the sun, which was once borne through the circle that is the Milky Way, created a path when it moved out of this orbit by scorching the region).

59. See McKirahan 1992: 242 and Johnson 2009: 336; τὰ φαινόμενα, however, includes not only perceptible objects such as the heavenly bodies but also λεγόμενα and ἔνδοξα. See below.

Aristotle's focus on the role of mathematics in the *pragmateia* of the "so-called" Pythagoreans raises an important question: if the *pragmateia* of the mathematical Pythagoreans involves application of mathematical principles to the objects of nature, how is this system distinguished from the *pragmateia* of the acousmatic Pythagoreans (if, indeed, that system is to be found in the *Metaphysics* at all)?

We can approach this problem by investigating Aristotle's descriptions of the first principles ascribed to the "so-called" Pythagoreans. In an earlier passage of *Metaphysics* A, where the "so-called" Pythagoreans appear for the first time in the text, we get a more precise description of what Aristotle took their principles to be:

> In the time of these men [i.e. Leucippus and Democritus] and before them[60] the "so-called" Pythagoreans were the first to latch onto mathematics. They advanced mathematics and, by being brought up in it, they began to believe that the principles of mathematics [ἀρχὰς τούτων [τῶν μαθημάτων]] were the principles of all things in existence [ἀρχὰς τῶν ὄντων πάντων]. And since numbers are first among these [i.e. beings][61] by nature, they seemed to see many resemblances [ὁμοιώματα] in numbers to things that are and things that come into being, rather than in fire and earth and water. For example: *this* attribute of numbers was justice, *that* was soul and mind, and another opportunity, and all the rest, so to speak, in the same way. Moreover, because they saw that the attributes and ratios of musical scales consisted in numbers—well, since other things seemed to be modeled [ἀφωμοιῶσθαι] on numbers in their nature in its entirety, and numbers seemed to be primary of all nature, they began to assume that the elements of numbers were the elements of things that are and that the whole of heaven was musical scale [ἁρμονία] and number [ἀριθμός]. Whatever resemblances to the attributes and regions of heaven and the entire order of the cosmos they were able to show [δεικνύναι] to be in numbers and musical scales, they collected [συνάγοντες] and fitted

60. Or, as Schofield (2012: 144) translates: "Among these thinkers." It is true that ἐν τούτοις could mean "among them," but it is difficult to square this with the temporal sense of πρὸ τούτων that follows. Alexander of Aphrodisias (*in Metaph.* p. 37.6–16 Hayduck) felt the need to explain this phrase as well, and he glosses: "Concerning the Pythagoreans, he says that some were born before Democritus and Leucippus, and some lived about the time of them (κατὰ τούτους)." He further explains that Pythagoras himself lived "a bit" (ὀλίγον) before Democritus and Leucippus, but that "many of the Pythagorean students (ἀκουσάντων) flourished at the same time as them."

61. See Schofield 2012: 144, with n. 8.

> them together [ἐφήρμοττον]. And if something were to be missing anywhere, they hastened to supplement it [προσεγλίχοντο][62] in order that their entire *pragmateia* might hang together. For example, since the number 10 is thought to be perfect and to encompass the entirety of numbers in their nature, they assert that there are ten things in heavenly orbit; but since there are only nine that are actually visible throughout the heavens, they invent a tenth, the Counter-Earth.
>
> We have treated this subject in greater detail elsewhere.[63] But the object of our discussion is to learn from them, too, what principles they posit, and how these correspond to the causes we have discussed. Well, then, evidently these men too believe that number is a principle, both in terms of matter for things that are and in terms of their attributes and states. And they take the elements of number to be the odd and the even, and, of these, the former to be limited [πεπερασμένον], and the latter unlimited [ἄπειρον]; the one is constituted of both of these (since it is both odd and even); number is derived from the one; and, as we've already said, the whole heaven is numbers.
>
> (ARISTOTLE, *Metaphysics*, 1.5, 985b23–986a21; translation after Schofield 2012)

This passage gives us a sense of what Aristotle thought the philosophical method and the first principles of the "so-called" Pythagoreans to be. In particular, it identifies what Aristotle understood to be a problem in their attempts to provide demonstrations.[64] According to Aristotle, the "so-called" Pythagoreans contaminate their understanding of the sensible facts by "hastening to supplement" whatever might be lacking in the empirical data with theoretical knowledge assumed by the "reason why"; within context of the classification of types of knowledge discussed earlier in *Metaphysics* A, it is not surprising that this

62. Schofield (2012: 144) has "they bent their efforts." This uncommon word appears at *Metaph.* 14.3, 1090b31, where Aristotle likewise complains that some Platonists (Xenocrates or Speusippus? See Annas 1976: 209–210) hasten to apply mathematics to the Forms (προσγλιχόμενοι ταῖς ἰδέαις τὰ μαθηματικά). At *Cael.* 2.13, 293a27, Aristotle also accuses some of the "so-called" Pythagoreans of "attracting the data to certain rationales and opinions of their own [πρός τινας λόγους καὶ δόξας αὑτῶν τὰ φαινόμενα προσέλκοντες]."

63. It has been discussed at *Cael.* 2.13, 293a18–b15, an extremely challenging passage that has presented many difficulties for scholars; it also appeared in Aristotle's lost writings on the Pythagoreans, especially in F 203 Ross, preserved by Alexander. For my analysis of this passage from *De Caelo*, see chapter 2, section entitled "Pythagoreanism and the Axiology of What Is 'Honorable.'"

64. See Schofield (2012: 153), who also notes that Aristotle does seem to ascribe to the Pythagoreans a logic.

characterization of "so-called" Pythagorean philosophical method obtains.[65] To put it simply, according to Aristotle, the "so-called" Pythagoreans adapt the immediate facts (τὸ ὅτι) to fit the explanation (τὸ διότι) in an ad hoc manner. The example given involves the bodies of the heavens: nine bodies can be perceived by the senses, but, since the Pythagoreans assume the number 10 as the perfect number, and since all things are number, there must be ten heavenly bodies. This example also reveals Aristotle's second substantial criticism of the philosophical system of the "so-called" Pythagoreans: they hastily and carelessly compare things in order to secure relationships between their first principles and observed phenomena. Such an activity leads the Pythagoreans, in Aristotle's estimation, to leave out the efficient and final causes. The "so-called" Pythagoreans' a priori philosophical methodology, which flies in the face of observation, is further described as a "fitting together" (ἐφήρμοττον), a word whose semantics are related both to investigation of the heavens elsewhere in Aristotle and to the concept of "musical scale" (ἁρμονία) more generally in Greek culture.[66] One might therefore hear an echo of a Pythagorean concept. Indeed, in Philolaus's work *On Nature* (F 7 Huffman = Stob. *Ecl.* 1.21.8), he claims that "the first thing fitted together [τὸ πρῶτον ἁρμοσθέν], the one in the center of the sphere, is called the hearth."[67] And as Carl Huffman has shown, Philolaus is to be included among the "so-called" Pythagoreans, and, moreover, that Aristotle had Philolaus chiefly in mind when criticizing their methodology.[68]

In pursuit of the metaphysics of Philolaus and the other "so-called" Pythagoreans, Aristotle returns to the interrogation of what is primary in their *pragmateia*. He claims that the "so-called" Pythagoreans posit the objects of mathematics as the first principles and elements of everything. The chief example he gives is number, which is apparently primary in two senses: as formal and material

65. Compare Arist. *Metaph.* 1.1, 981a11–32, on which see McKirahan 1992: 242.

66. See Arist. *MA* 1, 698a10–14, where ἐφαρμόττειν is translated by Nussbaum as "be in harmony." Lennox (2001a: 10 n. 23) would translate in a more decidedly methodological way as "apply" and notes that "Aristotle likely has in mind the application of universal accounts *via* proof to the particulars, since it was in order to understand them that the search for the universal began."

67. Luca Castagnoli points out to me that the language is the same, but that Aristotle's criticism—which emphasizes how a priori principles and phenomena are "fitted together" harmonically—diverges from the actual meaning of Philolaus's fragment.

68. Huffman 1993: 225, with reference to this passage, but without a sufficient discussion of Aristotle's description of "so-called" Pythagorean methodology. For a very good general analysis of the relationship between Philolaus's fragments and this passage, see Huffman 1993: 177–193.

cause.[69] Aristotle's concern with form and matter is reinforced by the description of those things that are primary as either first principles (ἀρχαί) or elements (στοιχεία). They are first principles with regard to their function as formal causes, but they are elements with regard to their function as the "that out of which" things in the universe are constructed.[70] These Aristotle defines according to a hierarchy based on priority: the odd and even are apparently prior to the one, since the one is constructed out of them, and the one is prior to number, since numbers are constituted from the one. And since all perceptibles have number as a property, they somehow derive from number itself.

This passage, however, stimulates us to consider whether another set of principles are ontologically prior to the odd and the even: limit and unlimited. After all, Aristotle thinks that the attributes of limit are present in the odd (i.e. it "has been limited" [πεπερασμένον]), whereas the even itself is unlimited (ἄπειρον).[71] A hierarchy of entities, even among first principles, is implicit in this passage, and the "so-called" Pythagoreans might be thought to attempt to provide explanations by means of demonstration that limit and unlimited are prior to odd and even. Apparently, the hierarchy is given its order on the assumption that the prior principles must act on those things given definition by them. In the case of limit and unlimited, we have good evidence from the genuine fragments of Philolaus of Croton that Aristotle, even if he was distorting Philolaus's thought, was essentially presenting a verifiable account of how some of the more sophisticated Pythagoreans undertook demonstration by employing mathematical objects as principles in demonstrations that involved perceptible objects.

One example of this approach to demonstration is a particularly difficult fragment of Philolaus preserved by Stobaeus:

69. See Zhmud 2012: 436–437. For an excellent exposition on number as material cause here, see Schofield 2012: 145–147.

70. The subject of the relationship between principles and elements is well-trodden ground, and I don't wish to pursue this question too far. It does not appear to me that, in *Metaphysics* A, Aristotle distinguishes explicitly between these, or that he has discovered a clear means of distinguishing them, but that does not mean that they are simply synonymous either. When he composed book Λ (12.6, 1071b22–26), he distinguished between first principles as external and elements as inherent. It is suggestive in book Δ (5.1, 1013a7–10) that he defines ἀρχή in several ways, among them (1) the thing as a result of whose immanent presence something first comes into being, and (2) that from which something comes into being, although it is not present in it. When defining στοιχεῖον in the same book (5.3, 1014a26–30), he clarifies that it is an immanent, indivisible entity out of which other things are composed and draws reference to the "elements of sound." Was Aristotle referring to Pythagorean mechanical attempts to obtain the basic elements of the concords here?

71. This is confirmed at *Metaph.* 1.8, 990a8–12.

It is necessary [ἀνάγκα] that the things that are be all either limiting [περαίνοντα], or unlimited [ἄπειρα], or both limiting and unlimited, but not in every case unlimited alone. Well then, since [ἐπεὶ] it is apparent [φαίνεται] that they are neither from limiting things alone, nor from unlimited things alone, it is clear, then [τᾶρα], that the cosmos and the things in it were fitted together [συναρμόχθη][72] from both limiting and unlimited things. Things in their activities also make this clear [δηλοῖ δὲ καὶ τὰ ἐν τοῖς ἔργοις]. For, some of them from limiting [constituents] limit, others from both limiting and unlimited [constituents] both limit and do not limit, others from unlimited [constituents] will appear to be unlimited.

(F 2 Huffman = Stobaeus, *Eclogae* 1.21.7a; translation after Huffman 1993)

Careful examination of this fragment demonstrates that Aristotle's criticism is not off the mark: Philolaus undertakes some sort of demonstration by reducing perceptibles to the objects of mathematics. This is already suggested in the first few lines and is quite explicit in the statement that one can see that limiters and unlimited things constitute the cosmos when one detects them in "activities" (τὰ ἔργα), a word that seems to refer to the attributes that we can perceive.[73] It is also implicit in the language used to discuss how things could be the way they are, as Philolaus uses particles (e.g. ἐπεί and τᾶρα) that suggest modal relations and appeals to philosophical concepts used commonly in Aristotelian demonstrations, such as necessity (ἀνάγκα).

If I am correct in thinking that the "so-called" Pythagoreans as described in Aristotle's *Metaphysics* A are one and the same as the mathematical Pythagoreans in his lost writings on the Pythagoreans, then we should expect to find a description of the *pragmateia* of the acousmatic Pythagoreans, who did not engage in demonstrations of some sort. Indeed, in a passage that immediately follows on the long passage (*Metaph.* 1.5, 985b24–986a21), our hypotheses are corroborated: we get a very concise description of what appears to be the *pragmateia* of the acousmatic Pythagoreans:

72. The appearance of this term, along with other terms related to ἁρμόζω, in other fragments of Philolaus (e.g. F 1 Huffman = D.L. 8.85: ἁ φύσις δ' ἐν τῷ κόσμῳ ἁρμόχθη ἐξ ἀπείρων τε καὶ περαινόντων; F 7 Huffman = Stob. *Ecl.* 1.15.7: τὸ πρᾶτον ἁρμοσθέν ἑστία καλεῖται) is suggestive evidence for the correlation between Aristotle's description of the demonstration of the "so-called" Pythagoreans, which involves "fitting together in addition" (ἐφήρμοττον) the "resemblances to the attributes and regions of heaven and the entire order of the cosmos."

73. On translating this difficult word, see also the account of Huffman 1993: 111–112.

People other than these very people [i.e. the "so-called" Pythagoreans][74] say [λέγουσιν] that there are ten principles, which they name in two elementary columns of cognates [τὰς κατὰ συστοιχίαν λεγομένας]:

Limit	Unlimited
Odd	Even
One	Plurality
Right	Left
Male	Female
Rest	Motion
Straight	Curved
Light	Darkness
Good	Evil
Square	Oblong . . .[75]

The Pythagoreans declared how many and what sorts [πόσαι καὶ τίνες] of contraries there were. Thus, from both of these authorities [i.e. Alcmaeon of Croton and the Pythagoreans] we can gather this much, that the contraries are the first principles of things in existence; but how many and what sorts these are [we can gather] from [only] one of these authorities [i.e. from the Pythagoreans]. Nevertheless, *how* [πῶς] these principles can be brought together [συνάγειν] and referred to our aforementioned list of causes has not been clearly articulated [σαφῶς οὐ διήρθρωται] by them, but they seem to arrange [ἐοίκασι τάττειν] the elements under the grouping of matter; for they say that substance is composed and fashioned out of these underlying elements.

(Aristotle, *Metaphysics* 1.5, 986a22–b8)

Aristotle, I suggest, seems to distinguish these "Pythagoreans" (as well as Alcmaeon of Croton) from the "so-called" Pythagoreans by appeal to their respective treatments of the first principles. The scientific pursuit of *these* "Pythagoreans" only goes so far as to (1) postulate the number and types of contraries, and (2) put them in an order. They put their principles in an order based on contrariness, and with no further attention to definition, nor any

74. Schofield (2012: 155–157) identifies this group as the "*sustoichia* theorists" and sees them as differentiated from the previously described group by the fact that "he ends up finding it difficult to ascribe any significant contribution from Alcmaeon and the *sustoichia* theorists to his current project." I suggest that the main reason for this is the fact that they do not obviously contribute to a science of demonstration.

75. Following Huffman (1993: 10–11), I have excised anything that deals explicitly and solely with Alcmaeon of Croton, whose status as a "Pythagorean" is questionable.

attempt to provide a demonstration for this organization. The activity of "arrangement" (τάξις), of course, occupies a significant role in Aristotle's philosophy, and, as he says at *Topics* 8.1, 155b9–10 (of the arrangement of questions), it is the activity peculiar to the dialectician, whose practice as such is contrasted against that of the philosopher, who engages in "demonstration" (ἀπόδειξις).[76] Such an arrangement cannot be considered a type of demonstration, nor is there any evidence of these "Pythagoreans" offering an explanatory "reason why" (τὸ διότι) the elements of their so-called Table of Contraries are arranged the way they are.[77] In the case of these "Pythagoreans," there is no attempt to show, for instance, *how* or *why* the limiter limits things in existence, or to provide an explanation for the systematization that is given.[78] To put it another way, the *pragmateia* associated with these "Pythagoreans" does not help a student to "grasp the demonstration as a demonstration, coming to see its premises *as* the causes and explanations of its conclusion."[79] From Aristotle's point of view, the "Table of Contraries" constitutes the sort of "perceptible" that falls under the umbrella term τὰ λεγόμενα: the "Table of Contraries" appears to function (for Aristotle's purposes) as data derived from observation (φαινόμενα).[80] It does not seem that Aristotle believes that the information given in the "Table of Contraries" listed here could be used as premises to generate demonstrations, even if it still has some residual value for Aristotle's own inquiry—otherwise it simply wouldn't be included. With regard to Aristotle's project in *Metaphysics* A, the Table itself functions as a sort of φαινόμενον in two ways: first, to the Pythagoreans who espouse it, it functions as a type of λεγόμενον, namely, what is passed down orally from Pythagorean teacher to student, an ipse dixit injunction like the *acusmata*. In this sense, the "Table of Contraries" does not represent anything other than the empirically derived "facts" that are immediate and familiar, at least for these "Pythagoreans." Second, for Aristotle himself, the Table and its

76. Arist. *Top.* 1.1, 100a25–31. This subject is, of course, a contentious point among scholars. But for my purposes, it serves only to exhibit Aristotle's attempt to distinguish two types of reasoning: that which proceeds by appeal to demonstration and that which proceeds by appeal to ordering.

77. For a useful treatment of "nondemonstrative" science as that which allows premises to multiply infinitely, see Smith 2009: 54.

78. See Huffman 1993: 47 n. 1.

79. As eloquently put by Robin Smith (1997: xvii).

80. I am adapting the famous argument of G. E. L. Owen (1986: 242–243) to include the opinions of previous philosophers in *Metaphysics* A as the sorts of ἔνδοξα or λεγόμενα that could be construed as φαινόμενα.

contents function as an ἔνδοξον, a reputable opinion that Aristotle is able to employ in the course of his own predemonstrative inquiry (ἱστορία).[81] Given Aristotle's lack of attribution of any sort of reasoning that involves demonstrations to these "Pythagoreans" and the implication that their "Table of Contraries" is to be considered a φαινόμενον, we can speculate with some reason that Aristotle considered these "Pythagoreans" to be the same as the acousmatic Pythagoreans discussed in his lost writings on the Pythagoreans.

There remains a third and final passage in *Metaphysics* A that refers to Pythagoreans of one or the other sort. Initially, it might seem unclear to which group Aristotle is referring. This text, I suggest, is also crucial to our understanding of Pythagoreanism, as reconstructed and appropriated by Aristotle, because it illuminates another way Pythagoreans engaged in their *pragmateia*, that is, through definitions:

> But while the Pythagoreans have claimed in the same way that there are two principles, they made this addition, which is peculiar to them, namely that they thought that the limited and the unlimited were not uniquely different substances[82], such as fire and earth and anything else of this sort, but that the unlimited itself and the one itself were the substance of the things of which they are predicated, and hence [διό] that number was the substance of all things. Concerning these issues, then, they expressed themselves in this way. And concerning essence [περὶ τοῦ τί ἐστιν], they began to make statements and definitions [λέγειν καὶ ὁρίζεσθαι], but their treatment was too simple [λίαν ἁπλῶς ἐπραγματεύθησαν]. For they both defined superficially and thought that the substance of the thing [ἡ οὐσία τοῦ πράγματος] was that to which a stated term would first be predicable, e.g. as if someone were to believe that "double" and "two" were the same because "two" is the first thing of which "double" is predicable. But surely to be "double" and to be "two" are not the same things. If that were to be the case, one thing would be many—a consequence that they actually drew.
>
> (ARISTOTLE, *Metaphysics* 1.5, 987a13–27)

While it is true that Aristotle refers to this group as "the Pythagoreans," and not the "so-called" Pythagoreans, it is nevertheless probable that this is a description

81. On the role of the "predemonstrative inquiry" in Aristotle's scientific works, see Lennox 2001b: 40–46.

82. Taking φύσεις in the sense later defined by Aristotle in book Δ (5.6, 1014b35–37) and only because it makes sense of Aristotle's use of the term "substance" (οὐσία) in the next sentence.

of a mathematical Pythagorean *pragmateia*.[83] This group of Pythagoreans is not simply listing first principles as contraries and assuming them as elemental to all things in existence. According to Aristotle, this group of Pythagoreans "began to make statements and definitions" and engaged in a primitive analysis concerning the essence (lit. the "what is" [*περὶ τοῦ τί ἐστιν*]), although their "treatment" (*ἐπραγματεύθησαν*, i.e., the application of their methods to the *pragmata* of their inquiry) was too simple. There also appears to be some preservation of an argumentative technique: these Pythagoreans thought that, since the "unlimited" and the "one" are the substance (*οὐσία*) of the things of which they are predicated, *therefore* number is the substance of all things. These are quite important innovations in philosophy for Aristotle, by contrast with the monists and pluralists, whose philosophy sought to describe the world without providing definitions by appeal to metaphysics and logic.[84]

The accumulation of evidence concerning the *pragmateia* of the mathematical Pythagoreans from *Metaphysics* A corroborates and further expands my two hypotheses, namely (1) that Iamblichus in his work *On the General Mathematical Science* 25 has excerpted a section from Aristotle's lost works on the Pythagoreans that accounts for the different *pragmateiai* of the mathematical and the acousmatic Pythagoreans, and (2) that those mathematical Pythagoreans described by Aristotle in his lost works on the Pythagoreans are the same as the "so-called" Pythagoreans of *Metaphysics* A and elsewhere in his texts.

MATHEMATICAL PYTHAGOREANISM AND THE "OBJECTS OF MATHEMATICS"

Given the detailed account above of the ways Aristotle distinguishes the *pragmateia* of the "so-called" Pythagoreans in *Metaphysics* A, we can now come back to Aristotle's account as preserved by Iamblichus in his work *On the*

83. There could be a very good reason for this. As Cherniss (1944: 192, with n. 112) suggests, this passage appears to have been inserted later by Aristotle. If, as I think, Aristotle only distinguished between the "so-called" (i.e. mathematical) Pythagoreans and the "Pythagoreans" (i.e. acousmatic) in his earlier treatments of the history of philosophy, which would include the crucial passage (1.5, 985b24–986b8) that demonstrates the differences, and if later on he only concerned himself with the philosophy of the mathematical Pythagoreans, then it would be unsurprising for him to refer to the mathematical Pythagoreans here as "Pythagoreans" *simpliciter*.

84. For a comprehensive analysis of this passage, now see Schofield 2012: 161–165. I follow Schofield in believing that Aristotle probably has Philolaus's F 6 directly in mind, but I also note the significance (again) of the term *ἡ οὐσία τοῦ πράγματος*, which may have a resemblance to Philolaus's *ἃ ἐστὼ τῶν πραγμάτων*. I will discuss the mathematical Pythagorean responses to predication further in chapters 4 and 5.

General Mathematical Science 25. Another revealing passage, also thought by Walter Burkert, Carl Huffman, Myles Burnyeat, and Oliver Primavesi to have been derived from Aristotle,[85] continues from the excerpt I have discussed:[86]

> (D) The Pythagoreans devoted themselves to mathematics. They both admired the accuracy of its arguments, because it alone among things that humans practice contains demonstrations [*εἶχεν ἀποδείξεις ὧν μετεχειρίζοντο*], and they saw that general agreement is given in equal measure to theorems concerning attunement, because they are [established] through numbers, and to mathematical studies that deal with vision, because they are [established] through diagrams. This led them to think that these things and their principles are quite generally the causes of existing things. Consequently, these are the sorts of things to which anyone who wishes to comprehend things in existence—how they are—should turn their attention, namely numbers and geometrical forms of existing things and proportions, because everything is made clear [*δηλοῦσθαι*] through them. So, then, by attaching the powers of each thing to the causes and primaries—only things that were less opportune or less honorable than them—they defined other things, too, in nearly the same manner. (E) Therefore, their education in numbers and the objects of mathematics [*τὰ μαθήματα τῶν πραγμάτων*] seemed to come through these subjects and in this general sketch. Such was also the method of demonstrations [*ἡ μέθοδος τῶν ἀποδείξεων*] among them, which both arose out of such principles and thereby attained fidelity and security in their arguments.
>
> (IAMBLICHUS, *On the General Mathematical Science* 25, 78.8–26)

The information preserved by Iamblichus in section (D) suggests that Aristotle, in his lost works on the Pythagoreans, continued to refer to the mathematical

85. See Burkert 1972: 50 n. 112, followed by Primavesi (2012: 251–252). Burkert (447–448), however, claims that Iamblichus or someone else has made spurious insertions in two places: "and to mathematical studies that deal with vision, because they are [established] through diagrams" and "and geometrical forms of existing things." Important correctives have been offered by Burnyeat (2005a: 38–43), who appeals to Arist. *APo.* 1.13, 79a7–8 in arguing that nothing should be excised here. This is in keeping with the stylistic traits of Iamblichus when he quotes from Aristotle, as recently analyzed by D. S. Hutchinson and Monte Ransome Johnson: he tends to preserve large blocks of material without modifying them (2005: 281–282). Zhmud (2007: 84–95) speculates without extensive direct evidence that Nicomachus is the source here; but even he admits that Nicomachus has nothing to say about demonstration, which is a central topic throughout *DCM* 25.

86. See above in the section entitled "Aristotle on the *Pragmateiai* of the Pythagoreans."

Pythagorean *pragmateia*. Indeed, there is good reason to believe, with Burnyeat, that Aristotle is criticizing specifically the activities of the mathematical Pythagorean Archytas of Tarentum here, although we should not assume that Archytas's philosophy is the only object of Aristotle's criticism.[87] It is also striking that, in section (E), Iamblichus refers to the educational curriculum of the mathematical Pythagoreans as dealing with the "objects of mathematics" (*τὰ μαθήματα τῶν πραγμάτων*), a peculiar phrase that is unattested anywhere else in Iamblichus's oeuvre, or, for that matter, in what remains of Greek philosophy or mathematics. We do, however, see something very close to it in Aristotle's description in *Metaphysics* A of the *pragmateia* of Plato,[88] which is considered a successor to the philosophical *pragmateia* of the "Italians,"[89] although with some modifications:

> Therefore, Plato named these other sorts of entities "Ideas," and he [said that] perceptibles are all called after them and in accordance with them. For the many things that bear the same name as the forms exist by virtue of participation [*κατὰ μέθεξιν*] in them.[90] With regard to participation, he changed the name only: for whereas the Pythagoreans claim that objects in existence exist by way of imitation of numbers [*μιμήσει τῶν ἀριθμῶν*], Plato says by way of participation [*μεθέξει*], modifying the name. As to what participation or imitation is, however, they left it to us to seek it out together.
>
> Furthermore, Plato claims that in addition to perceptibles and Forms is a middle type of entity, the objects of mathematics [*τὰ μαθηματικὰ τῶν πραγμάτων*], which differ from perceptibles in being eternal and immutable, and from Forms in that many [objects of mathematics] are similar, whereas each Form itself is unique.
>
> (Aristotle, *Metaphysics* 1.6, 987b7–18)

87. Burnyeat 2005a. Contra Huffman (2005: 568), who thinks that Aristotle could not have referred to Archytas as a "Pythagorean." Given the explicit reference to the use of visual diagrams, we should also consider admitting figures like Eurytus of Tarentum, whose approach to definition of objects by means of pebble arithmetic was known to Aristotle. On Eurytus, see chapter 4, section entitled "Growing and Being: Mathematical Pythagorean Philosophy before Plato."

88. Identified explicitly as such at *Metaph.* 1.6, 987a30.

89. Arist. *Metaph.* 1.6, 987a29–31. Aristotle there draws comparisons with Plato and the Italians, although he more generally states that Plato succeeded the "aforementioned philosophies" (*μετὰ τὰς εἰρημένας φιλοσοφίας*). He is somewhat unclear here, but in regard to the inheritance of modes of definition given at *Metaph.* 13.4.3, 1078b17–23, Aristotle explicitly lists those who influenced Plato's inquiry into essence as Socrates, Democritus, and the "earlier" (*πρότερον*) Pythagoreans.

90. This is a notoriously difficult passage. I have adopted the text of Ross.

This is one of the more problematic passages in the history of ancient philosophy, and the task to identify with precision the objects of mathematics, as intermediaries between Plato's Forms and perceptibles, is not made easier by Aristotle's admitted confusion.[91] Part of the problem here is that the term *τὰ μαθηματικὰ τῶν πραγμάτων*, like the explicit ascription of a theory of imitation (*μίμησις*) *in numbers* to the Pythagoreans, is an Aristotelian construction that cannot be found anywhere in ancient philosophy outside Aristotle and his immediate associates.[92] It is not clear from this passage whether Aristotle would consider *τὰ μαθηματικὰ τῶν πραγμάτων* to be distinguished from other terms he uses to describe the objects of mathematics, especially the relatively common simple formulation *τὰ μαθηματικά*, which he uses often in reference to the ontological theories of Plato, Speusippus, and Xenocrates.[93] We have seen, of course, that Aristotle mentions the "objects of the mathematics" (*τὰ μαθηματικὰ τῶν ὄντων*) by reference to the first principles of the "so-called" Pythagoreans (*Metaph.* 1.8, 989b32). Generally, Aristotle does not seem to distinguish between *τὰ πράγματα* and *τὰ ὄντα* in referring to the "things"

91. See Ross's useful discussion of the problems that arise from this passage and for a history of their treatment from antiquity to the early twentieth century (1924: 161–168). Jaeger concerned himself with the principle of intermediary, without focusing on the objects of mathematics (1948: 91, with n. 2). Cherniss (1944: 75–78) denied that the objects of mathematics as intermediates existed for Plato and considered the ascription of this by Aristotle possibly to have been a misunderstanding of a passage from Plato's *Republic* (551a–e). Tarán (1981: 23 n. 120) followed Cherniss but saw the ascription of intermediary objects of mathematics to Plato as a point of contrast to Speusippus's postulation of separate and unchangeable mathematical numbers/ideas and magnitudes. Burkert (1972: 43–45) plausibly connects "imitation" to Aristotle's descriptions of "resemblances" (*ὁμοιώματα*) at *Metaph.* 1.5, 985b27 and concludes that "again and again it becomes clear that the Pythagorean doctrine cannot be expressed in Aristotle's terminology." Denyer (2007: 302–304) has argued in favor of the presence of intermediate mathematicals in Plato's epistemology but without reference to Pythagoreanism. Most recently, Steel (2012: 183) has aptly noted: "if some (as Cherniss) may complain about an excessive Pythagorising of Plato, one can as well point to a Platonisation of the Pythagorean doctrine of numbers."

92. *τὰ μαθηματικὰ τῶν πραγμάτων* occurs nowhere among the Peripatetic fragments. Aristoxenus (F 23 Wehrli) speaks of Pythagoras "likening all things to numbers" (*πάντα τὰ πράγματα ἀπεικάζων τοῖς ἀριθμοῖς*), on which see chapter 2. Theophrastus ascribes to Plato and the "Pythagoreans" a theory of *μίμησις*, in which sensibles within the universe are understood to imitate the first principles (*Metaph.* 11a26–11b7). But, as I've shown elsewhere (Horky, forthcoming), this theory should be ascribed to Xenocrates or, at most, to the "Pythagoreans" as seen through Xenocrates's point of view.

93. E.g. *Metaph.* 8.1, 1042a11–12; 12.1, 1069a35; 13.1, 1076a33; 13.2, 1077a16; 13.3, 1077b33, etc. Aristotle will speak of mathematicals that are "separate from" + genitive (e.g. *Metaph.* 13.2, 1076a33–34: *κεχωρισμένα τῶν αἰσθητῶν*) or "intermediate of" + genitive (e.g. *Metaph.* 11.1, 1059b6: *μεταξύ τε τῶν εἰδῶν καὶ τῶν αἰσθητῶν*).

that exist, but the unusual complication of τὰ μαθηματικά with either τὰ πράγματα or τὰ ὄντα in the genitive plural is a peculiarly Aristotelian formulation, and, moreover, is localized to discussions of Plato or the mathematical Pythagoreans (and, importantly, not Speusippus or Xenocrates) in *Metaphysics* A. As it turns out, in fact, the relatively unusual formulation τὰ μαθηματικὰ τῶν πραγμάτων most closely resembles the language of the mathematical Pythagorean Philolaus of Croton (F 6 Huffman = Stob. *Ecl.* 1.21.7d), who, when he spoke of the "being of things" (ἁ ἐστὼ τῶν πραγμάτων), was referring to the entity by virtue of which limiters and unlimiteds, the mathematical principles of his philosophy, could be thought to exist.[94] It is therefore probable that Iamblichus, in mentioning τὰ μαθήματα τῶν πραγμάτων in passage (B) from *On the General Mathematical Science*, was still looking at Aristotle's treatises on the Pythagoreans, perhaps written contemporaneously with *Metaphysics* A and, importantly, earlier than the treatments of the Pythagoreans in M or N. This is significant, because it suggests that it was Aristotle who celebrated the mathematical Pythagoreans for having achieved some credibility in their method of demonstration, even if they were overzealous in their pursuit of a unified philosophical system.

CONCLUSIONS

We have seen that the fundamental difference between acousmatic and mathematical Pythagoreanism as formulated by Aristotle lies in the latter group's attempts to make use of some sorts of demonstrative argumentation in order to provide explanations for their ideas. While acousmatic Pythagoreans apparently made no attempts to engage in demonstrations, mathematical Pythagoreans engaged in investigations that employed the principles of mathematics in order to make sense of the world they experienced. Their demonstrations tended to be derived from the principles of mathematics, including limiter and unlimited, as attested in the genuine fragments of Philolaus of Croton. It is also possible that their demonstrations were axiomatic and took the form of diagrams, as in the case of the speculative optical theories of Archytas of Tarentum.[95] Doubtless other types of Pythagorean demonstration have been lost to

94. The term ἁ ἐστὼ τῶν πραγμάτων has been used as grounds for dismissal of this fragment as authentic, especially since the term itself is replicated in the spurious Περὶ ἀρχων of Ps.-Archytas. But the authenticity of Philolaus's fragment has also been defended in various ways by Nussbaum (1979: 101) and Huffman (1993: 131–132). The term ἡ τῶν πραγμάτων οὐσία, which appears in Plato's *Cratylus*, also occurs by reference to Philolaus, as I argue in chapter 4, section entitled "Plato and Mathematical Pythagorean 'Being' before the *Phaedo*."

95. See Burnyeat 2005a: 45–51.

us. The plurality of the objects of their study made it difficult for Aristotle to characterize their philosophical system and to locate it squarely within the inquiry portion of his history of philosophy. The mathematical Pythagoreans were apparently also prone to establish relationships of similarity between numbers and perceptibles. What is more, as I will show, they posited an ontological order that was based on attributes that were strongly related to social organization within the *polis*, such as the notion of "what is more honorable," thus suggesting an organic relationship between the terms of political order and of ontological hierarchy. This important aspect of Aristotle's description of the *pragmateia* of the mathematical Pythagoreans is the subject of the first portion of chapter 2.